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FINAL REPORT

on

Contract NAS8-28019

ACTIVE CONTROL OF PRIMARY MIRROR OF AN ORBITING TELESCOPE WITH THERMAL EXCITATION

by

James L. Hill and John N. Youngblood
Co-Principal Investigators

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center

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I. INTRODUCTION

This report summarizes the work done on the extension of Contract NAS8-28019 during the period of January - December, 1973. The presentation here continues that of the Interim Report [1]* on the same contract published May, 1973. Much of the background for this report is contained in the Interim Report.

Briefly, the Interim Report presented feasibility results for smoothing surface irregularities of a homogeneous mirror by using a controlled pattern of heating units on the rear of the mirror. It was shown that significant deflections of the mirror surface could be caused. However, no actual error-suppression simulations were made.

This report presents the generalization that was made to model a layered structure of a kind that represents a light-weighted mirror. This theory is contained in Chapter II. In addition, the strategy for error suppression is derived in Chapter III. The results of a variety of error-suppression studies are presented in Chapter IV. The computer programs for all parts of this study are contained in Appendix A and Appendix B.

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* Numbers in brackets indicate entries in REFERENCES.

II. THERMAL RESPONSE ANALYSIS

The thermal response model of the primary mirror is similar to the model that was developed and presented in the Interim Report [1] of this project. The model was generalized to allow the primary mirror to be a three-layer structure as sketched in Figure II-1. The thermal and mechanical properties of each layer are orthotropic. The three orthotropic layers of the primary mirror represent the front, core and back. The orthotropy of each layer (particularly the core and back) is included to account for the directionality of the equivalent mechanical and thermal properties induced by the light-weighting of the mirror.

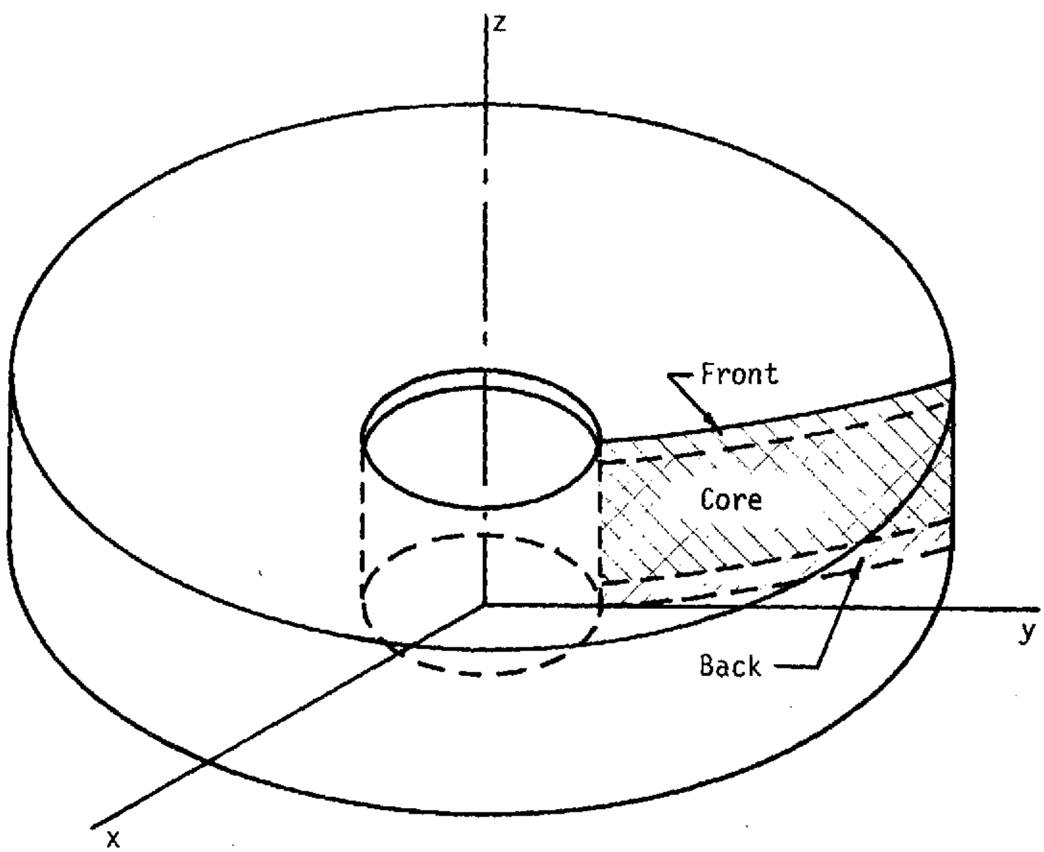


Figure II-1 Layered Model of Primary Mirror

Thermal Properties of Each Layer

The thermal properties of the i-th layer (i=1, 2, 3) are the following:

C - Specific thermal capacity which is equal to the specific heat times the mass density.

k_r, k_θ, k_z - Thermal conductivities in the r, θ, z- directions.

α_r, α_θ, α_z - Coefficients of linear thermal expansion in the r, θ, z- directions.

These thermal properties are constant within each layer.

Mechanical Properties of Each Layer

The mechanical properties of each orthotropic layer are given as the stress-strain relations given in [2].

$$
\begin{aligned}
\varepsilon_r &= (1/E_r)\sigma_r - (\nu_{\theta r}/E_\theta)\sigma_\theta - (\nu_{zr}/E_z)\sigma_z \\
\varepsilon_\theta &= -(\nu_{r\theta}/E_r)\sigma_r + (1/E_\theta)\sigma_\theta - (\nu_{z\theta}/E_z)\sigma_z \\
\varepsilon_z &= -(\nu_{rz}/E_r)\sigma_r - (\nu_{\theta z}/E_\theta)\sigma_\theta + (1/E_z)\sigma_z
\end{aligned} \qquad \text{(II-1)}
$$

$$
\begin{aligned}
\gamma_{r\theta} &= (1/G_{r\theta})\tau_{r\theta} \\
\gamma_{rz} &= (1/G_{rz})\tau_{rz} \\
\gamma_{\theta z} &= (1/G_{\theta z})\tau_{\theta z}
\end{aligned} \qquad \text{(II-2)}
$$

where

σ_r, σ_θ, σ_z = normal stresses

$\tau_{r\theta}$, τ_{rz}, $\tau_{\theta z}$ = shear stresses

ε_r, ε_θ, ε_z = normal strains

$\gamma_{r\theta}$, γ_{rz}, $\gamma_{\theta z}$ = shear strains

and the elastic properties E_r, E_θ, E_z, $\nu_{\theta r}$, ..., $\nu_{\theta z}$ must satisfy the relations

$$E_r \nu_{\theta r} = E_\theta \nu_{r\theta},\ E_\theta \nu_{z\theta} = E_z \nu_{\theta z},\ E_z \nu_{rz} = E_r \nu_{zr} \tag{II-3}$$

The Discrete Model of the Primary Mirror

The temperature and the displacement components are expressed in Fourier series forms. The Fourier coefficients of these four fields vary over the meridional (r,z) section. The (r,z) variation is discretized by the finite element method. This method of discretizing the temperature $T(r,\theta,z,t)$ and the displacements $u_r(r,\theta,z)$, $u_\theta(r,\theta,z)$ and $u_z(r,\theta,z)$ is exactly the same as previously used in our Interim Report [1]. The expressions for the temperature and the displacement fields are

$$T(r,\theta,z,t) = \sum_{n=-N}^{N} T_n(r,z,t)e^{i2n\pi\theta} \tag{II-4}$$

$$u_r(r,\theta,z,t) = \sum_{n=-N}^{N} U_n(r,z,t)e^{i2n\pi\theta} \tag{II-5}$$

$$u_\theta(r,\theta,z,t) = \sum_{n=-N}^{N} V_n(r,z,t)e^{i2n\pi\theta} \tag{II-6}$$

$$u_z(r,\theta,z,t) = \sum_{n=-N}^{N} W_n(r,z,t)e^{i2n\pi\theta} \tag{II-7}$$

where the component fields T_n, U_n, V_n, W_n are such that T, u_r, u_θ and u_z are real valued functions.

Each of the component fields is discretized over the meridional section by assuming linear variation over arbitrary triangular areas. The section is divided into a collection of triangles as sketched below in Figure II-2.

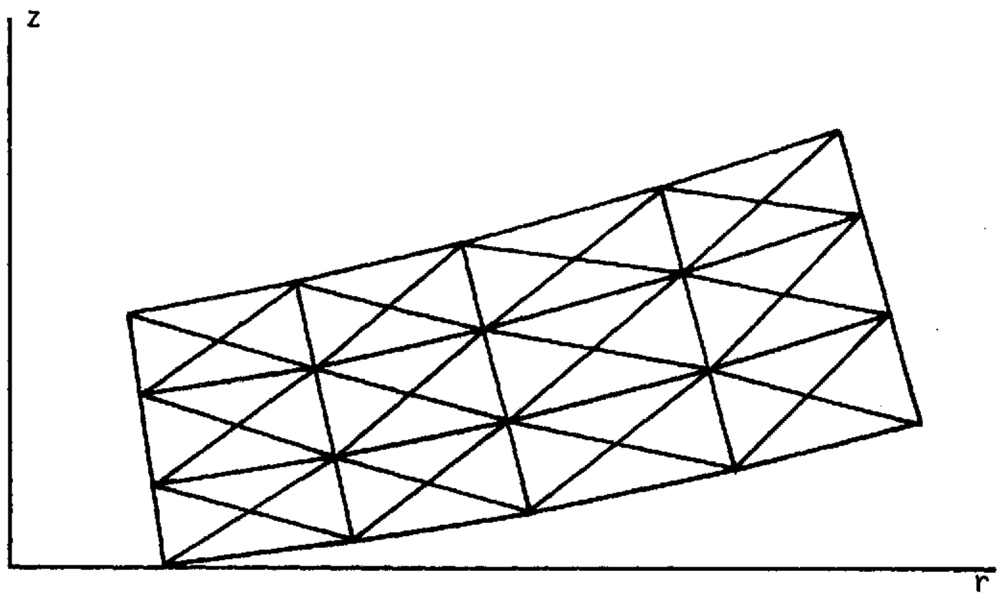


Figure II-2 Meridional Section of the Mirror

Consider any one of the triangular areas as sketched below.

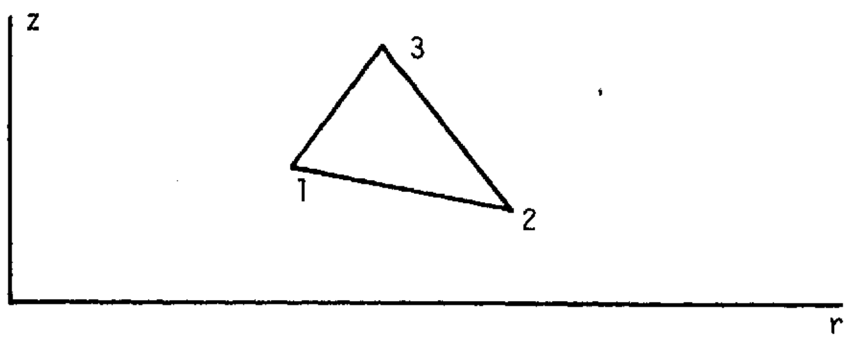


Figure II-3 Triangular Area

The component temperature $T_n(r,z,t)$ is represented as

$$T_n(r,z,t) = [\phi_,(r,z),\phi_2(r,z),\phi_3(r,z)]\underset{\sim}{T}_n(t) \qquad \text{(II-8)}$$

where $\underset{\sim}{T}_n(t)$ is a matrix of the nodal values of the component temperature

$$\underset{\sim}{T}_n(t) = [T_{n1}(t),\ T_{n2}(t1,T_{n3}(t)]^t \qquad \text{(11-9)}$$

and $[c]^t$ indicates the transpose of $[c]$. The interpolating functions $\phi_i(r,z)$ are given by

$$\phi_i(r,z) = a_i + b_i r + c_i z \tag{II-10}$$

where

$$\begin{aligned} a_1 &= (r_2 z_3 - r_3 z_2)/A_t \\ b_1 &= (z_2 - z_3)/A_t \\ c_1 &= (r_3 - r_2)/A_t \end{aligned} \tag{II-11}$$

$$A_t = [(r_2 - r_1)(z_3 - z_1) - (r_3 - r_1)(z_2 - z_1)]/2 \tag{II-12}$$

for i = 2 and 3 permute the indices.

Likewise the component displacements are represented as

$$\begin{Bmatrix} U_n(r,z,t) \\ V_n(r,z,t) \\ W_n(r,z,t) \end{Bmatrix} = [\phi_1(r,z)\underline{\underline{I}},\ \phi_2(r,z)\underline{\underline{I}},\ \phi_3(r,z)\underline{\underline{I}}]\ \underline{U}_n(t) \tag{II-13}$$

where $\underline{\underline{I}}$ is a 3x3 identity matrix and $\underline{U}_n(t)$ is a matrix of the nodal values of the component displacements given by

$$\underline{U}_n(t) = [U_{n1}(t),\ V_{n1}(t),\ W_{n1}(t),\ U_{n2}(t),\ \ldots,\ W_{n3}(t)]^t \tag{II-14}$$

Utilizing the discretizations of equations (II-8) and (II-13) over all the triangular areas that comprise the meridional section (r,z - section) of the mirror, the component temperature and displacements can be characterized by the values they have at the nodes of the section.

The temperature and displacements are governed by sets of partial differential equations and appropriate boundary conditions. Alternatively, the temperature and displacements are formulated by variational principles that are generalizations of those given in the Interim Report [1]. The generalizations involve accounting for the

orthotropic nature of the layers of the mirror. The development differs so little from that of the Interim Report that it will not be repeated here.

The finite element model of the component temperature results in a set of matrix equations of the form

$$\underline{\underline{C}}_n \, \underline{T}_n + \underline{\underline{D}} \, \dot{\underline{T}}_n = \underline{q}_n \tag{II-15}$$

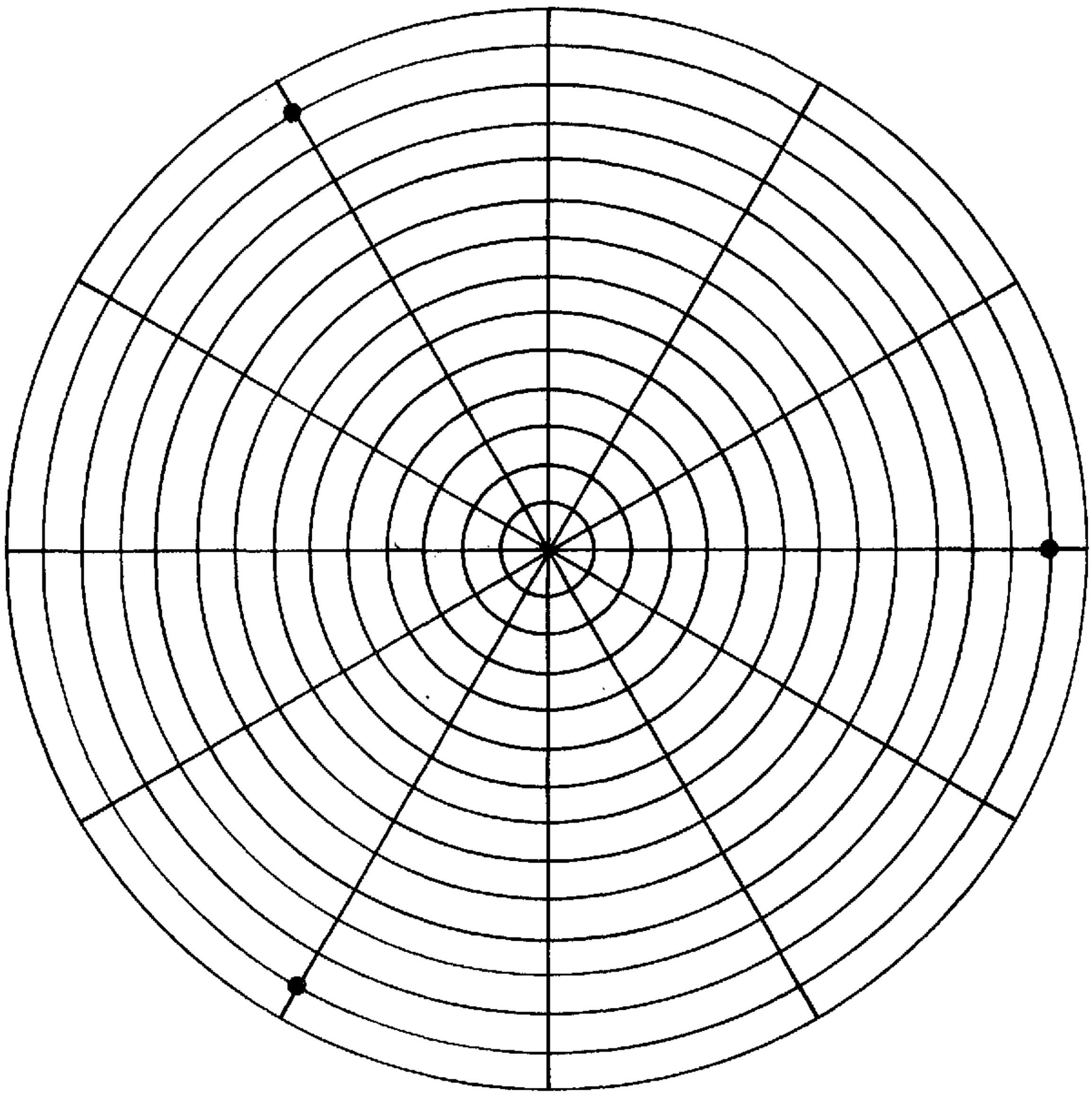
for each Fourier component. Similarly, the component displacements are governed by

$$\underline{\underline{K}}_n \, \underline{U}_n + \underline{\underline{P}}_n \, \underline{T}_n = 0 \tag{II-16}$$

Thus equation (II-15) is integrated for each set of nodal components $\underline{T}_n(t)$ which are substituted into equations (II-16). Equations (II-16) are solved by matrix inversion for $\underline{U}_n(t)$. The nodal components are substituted into equations (II-8) and (II-13) to obtain $T_n(r,z,t)$, $U_n(r,z,t)$, $V_n(r,z,t)$, and $W_n(r,z,t)$ which are then substituted into equations (II-4), (II-5), (II-6) and (II-7) to yield $T(r,\theta,z,t)$, $u_r(r,\theta,z,t)$, $u_\theta(r,\theta,z,t)$, and $u_z(r,\theta,z,t)$.

The component heat input $\underline{q}_n$ comes from the heating that is applied to the rear of the mirror. To characterize the steady-state thermoelastic response of the mirror, the normal displacements at a set of points of the front surface of the mirror due to placing a unit heater over a patch of the back were calculated.

The points on the front surface (the sample set) are at the intersections of the circles and radial lines shown in Figure II-4. The heater patches are between the concentric circles and are centered about the radial lines as shown in Figure II-5. The indices of a heater is the index of the inner circle and of the radial line. Let I be the



● - Support Points

Figure II-4 Sample Point Locations

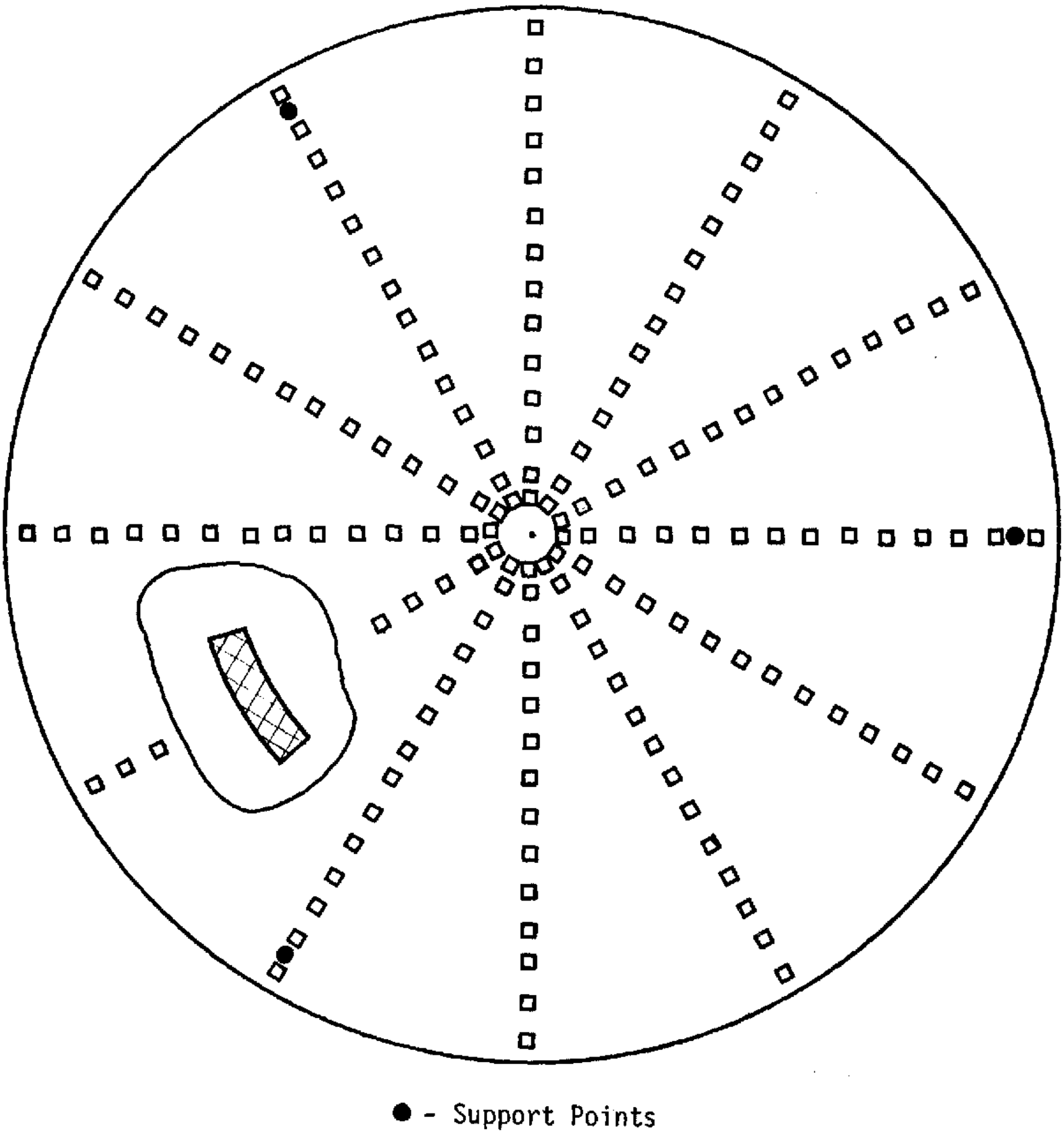


Figure II-5 Heat Patch Locations with a Typical Patch

index of the concentric circles (I = 1 is the inner boundary and I = IM is the outer boundary). The radial lines are indexed by K (K=1 is for θ=0, K=KM for θ=(KM-1)/KM*360°). The sample points are numbered from 1 to IM x KM. A particular sample point is identified by the index ISP given by

$$\mathrm{ISP} = (\mathrm{K}-1)\mathrm{IM} + \mathrm{I} \qquad \begin{matrix} \mathrm{I} = 1,2,\ldots, \mathrm{IM} \\ \mathrm{K} = 1,2,\ldots, \mathrm{KM} \end{matrix} \tag{II-17}$$

Likewise the heater patches are numbered from 1 to (IM-1)xKM. The index IHP of a heater is given by

$$\mathrm{IHP} = (\mathrm{I}-1)\mathrm{KM} + \mathrm{K} \qquad \begin{matrix} \mathrm{I} = 1,2,\ldots,\mathrm{IM}-1 \\ \mathrm{K} = 1,2,\ldots,\mathrm{KM} \end{matrix} \tag{II-18}$$

The Thermoelastic Influence Matrix

Define the steady-state normal displacement of the front surface at i due to a unit heat patch at j as a_{ij}. The displacements $\underline{w}$ at the sample points due to heat inputs $\underline{q}$ at the heater patches are given by the matrix equation

$$\underline{w} = \underline{\underline{A}}\underline{q} \tag{II-19}$$

For the calculations carried out in this study, IM = 15 and KM = 12 so that there were 180 sample points and 168 heater patches. This influence matrix $\underline{\underline{A}}$ represents the statical thermoelastic response of the mirror.

III. ANALYSIS OF ERROR SUPPRESSION

Introduction

The influence matrix which is computed from the structural response program may be used to associate a controlled deflection of the sample point set to a set of heat inputs on the back face of the mirror. The purpose of the control program is to compute the amount of heat input to a designated pattern of heat patches that reduces an arbitrary surface error to its minimum in an r.m.s. sense. The control program also computes the surface error (r.m.s.) before and after compensation. The r.m.s. error is interpreted as the root mean squared difference of sample point location of the mirror surface and the best-fit-sphere of the surface passing through the rigid support point.

Removal of Best Fit Sphere

In heating the mirror for control purposes, a significant amount of heat goes into the free thermal expansion of the mirror. This deflection does not contribute to local error smoothing and may be compensated by refocusing of the mirror. Moreover, if retained as part of the control, it significantly reduces the sensitivity of the control function. For these reasons the free thermal expansion terms are measured and their effect is removed from the influence matris.

Consider the mirror geometry shown in Figure III-1, which shows a spherical surface of radius R_t that is the best fit sphere through a set of displacements w_{ij} measured from the perfect sphere of radius R and also passes through the support points of the mirror.

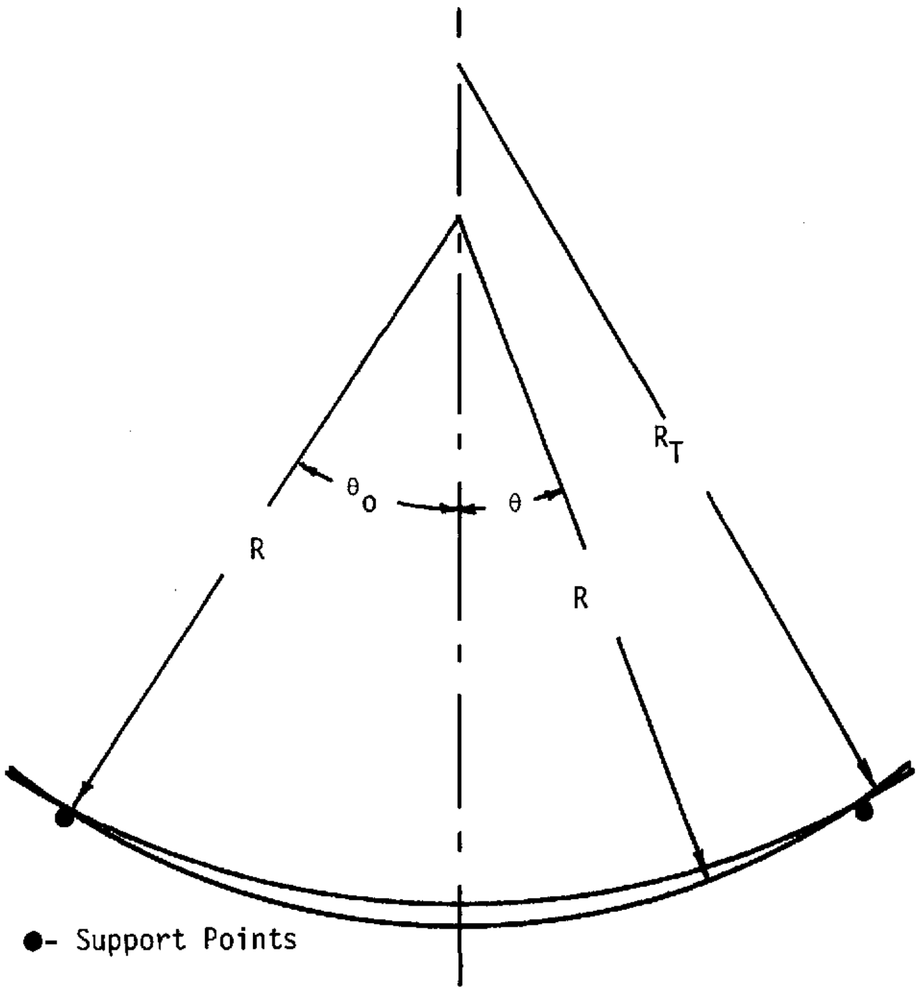


Figure III-1 Geometry of Best-Fit-Sphere

The difference in the two reference spheres is

$$y = (R_T - R)\left[\frac{\cos\theta}{\cos\theta_o} - 1\right]$$

The best fit sphere determined by the radius R_T is found by minimizing the functional

$$J = \sum_{i=1}^{IM}\sum_{j=1}^{JM}(w_{ij} - y_i)^2$$

where w_{ij} is the displacement of the $ij^{\underline{th}}$ surface nodal point due to heat input on the rear and y_i is the difference at θ_i.

The minimization yields

$$\Lambda = R_T - R = \sum_{i=1}^{IM}\sum_{j=1}^{IM} w_{ij}\left[\frac{\cos\theta_i}{\cos\theta_o} - 1\right] \Big/ JM\sum_{i=1}^{IM}\left[\frac{\cos\theta_i}{\cos\theta_o} - 1\right]^2$$

which represents the adjustment of the reference sphere.

The influence matrix as computed in the response program represents the correspondence of sample point deflection to heat input. Since the mirror may be focused to remove spherical errors, that part of the deflection which may be considered a readjustment of the reference sphere may be ignored. Therefore each column of the influence matrix (corresponding to one heat patch) may have its best-fit-sphere of influence removed. This is performed on each column of the influence matrix in the control program by application of (III-1). The modified influence matrix represents the influence of heater patches on sample point deflections about the resulting best-fit-sphere.

When the sample point uncompensated error vector is computed for the particular error under consideration, its best-fit-sphere is similarly removed. This corresponds to re-focusing the mirror prior to error suppression. The process is carried out in the control program.

In addition, the control program reduces the modified influence matrix to the appropriate array for the sample set and heat patch set selected.

Error Computation and Suppression

The deflection of the front of the mirror about its best-fit-sphere is composed of a disturbance term $\underline{w}_d$, whose effect is to be minimized and a controlled deflection term $\underline{w}_c$, imposed to reduce the effect of $\underline{w}_d$. Both $\underline{w}$'s represent a vector of sample point displacements.

In the control program the modified and reduced influence matrix $\underline{\underline{A}}$ is used to associate the heat patch input vector $\underline{q}$ to the controlled deflection $\underline{w}_c$.

$$\underline{w}_c = \underline{\underline{A}}\underline{q}$$

The total deflection vector is

$$\underline{w} = \underline{w}_d + \underline{\underline{A}}\underline{q}$$

The purpose of the heater pattern is to produce a vector $\underline{q}$ that minimizes the r.m.s. error

$$J = \sqrt{(\underline{w}^t\underline{w})/N}$$

where N is the number of sample points. It is understood that the modified influence matrix and modified disturbance vector are used so that the resulting error is measured about the best-fit-sphere.

The minimization of the r.m.s. error is accomplished by

$$\underline{q} = -\ (\underline{\underline{A}}^t\underline{\underline{A}})^{-1}\ \underline{\underline{A}}^t\underline{w}_d$$

This distribution of heats on the rear of the mirror yields a compensated r.m.s. error of

$$J_c = \sqrt{\underline{w}_d{}^t[\underline{\underline{I}} - \underline{\underline{A}}(\underline{\underline{A}}^t\underline{\underline{A}})^{-1}\ \underline{\underline{A}}^t]\underline{w}_d/N}$$

or a reduction in J^2 of

$$J^2 - J_c{}^2 = \underline{w}_d{}^t\underline{\underline{A}}(\underline{\underline{A}}^t\underline{\underline{A}})^{-1}\ \underline{\underline{A}}^t\underline{w}_d/N$$

The resulting surface error figure for a particular disturbance was normalized by the uncompensated error for the same disturbance. The resulting r.m.s. error figure is

$$\hat{J} = \sqrt{\frac{\underline{w}_d^{\,t}(\underline{\underline{I}} - \underline{\underline{A}}(\underline{\underline{A}}^t\underline{\underline{A}})^{-1}\ \underline{\underline{A}}^t)\underline{w}_d}{\underline{w}_d^{\,t}\ \underline{w}_d}}$$

Heat Input and Deflection Measurement Points

The node distribution for both the front and the rear of the mirror is shown in Figure II-4 and II-5. The surface has IM concentric nodal rings and KM nodal rays. The nodes occur at intersections of rings and rays. The nodes are selected by the program after mirror dimensions, KM, and IM are given.

The nodes for measurement of surface deflection are chosen in the control program. Any subset of the full set of nodes may be chosen.

The heat is applied to the rear by patches. The patches extend the entire distance between rings and make an angle PAN centered about a ray. Patch location and patch angle may be inputed in the control program.

To examine the feasibility of error suppression, the maximum set of sample points was used in the runs. Heater distributions comprising from 6 to 48 heat patches were used. The arrays of the heat patches are shown in Figures III-2 through III-6.

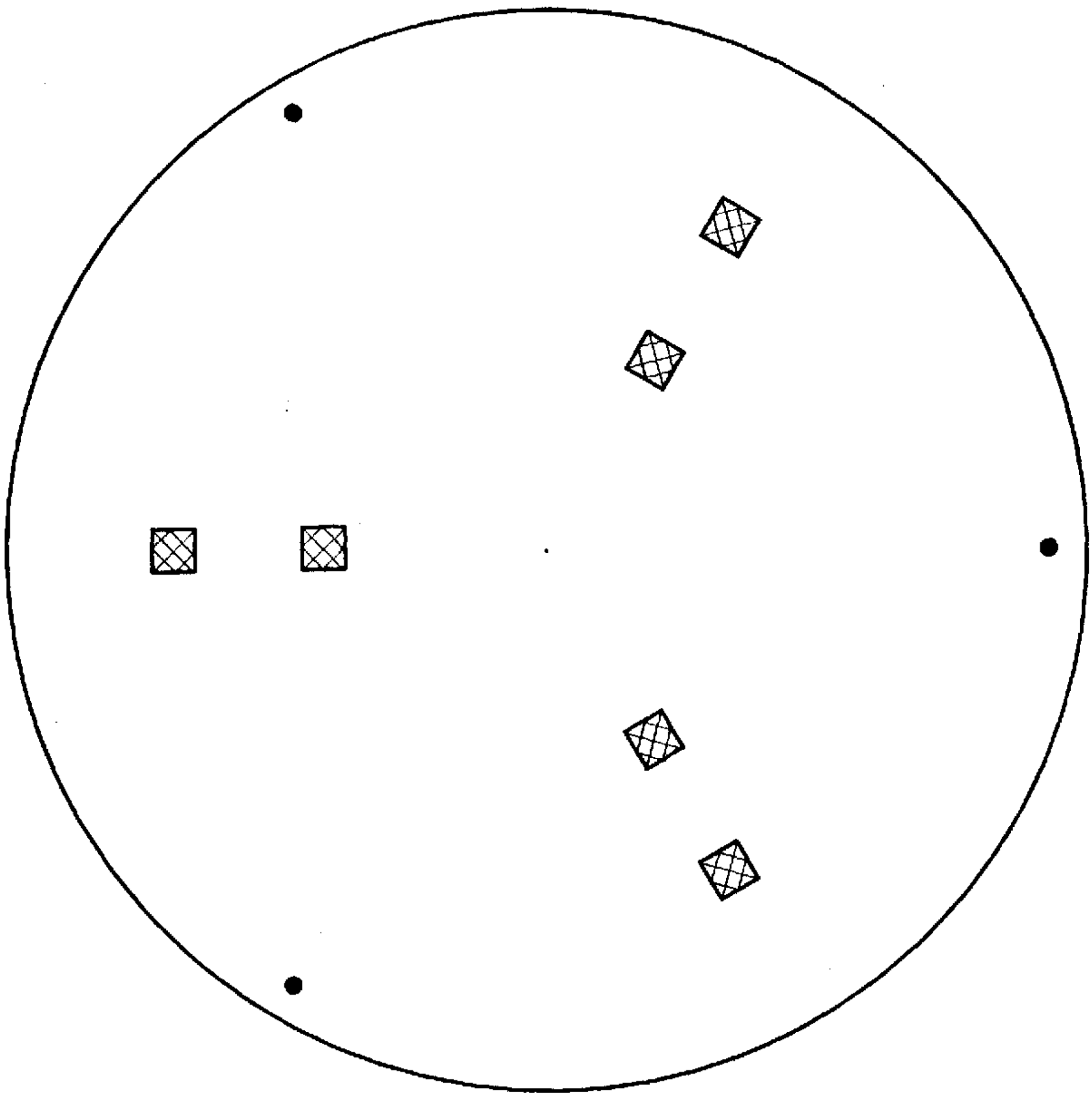


Figure III-2 Array of 6 Heat Patches

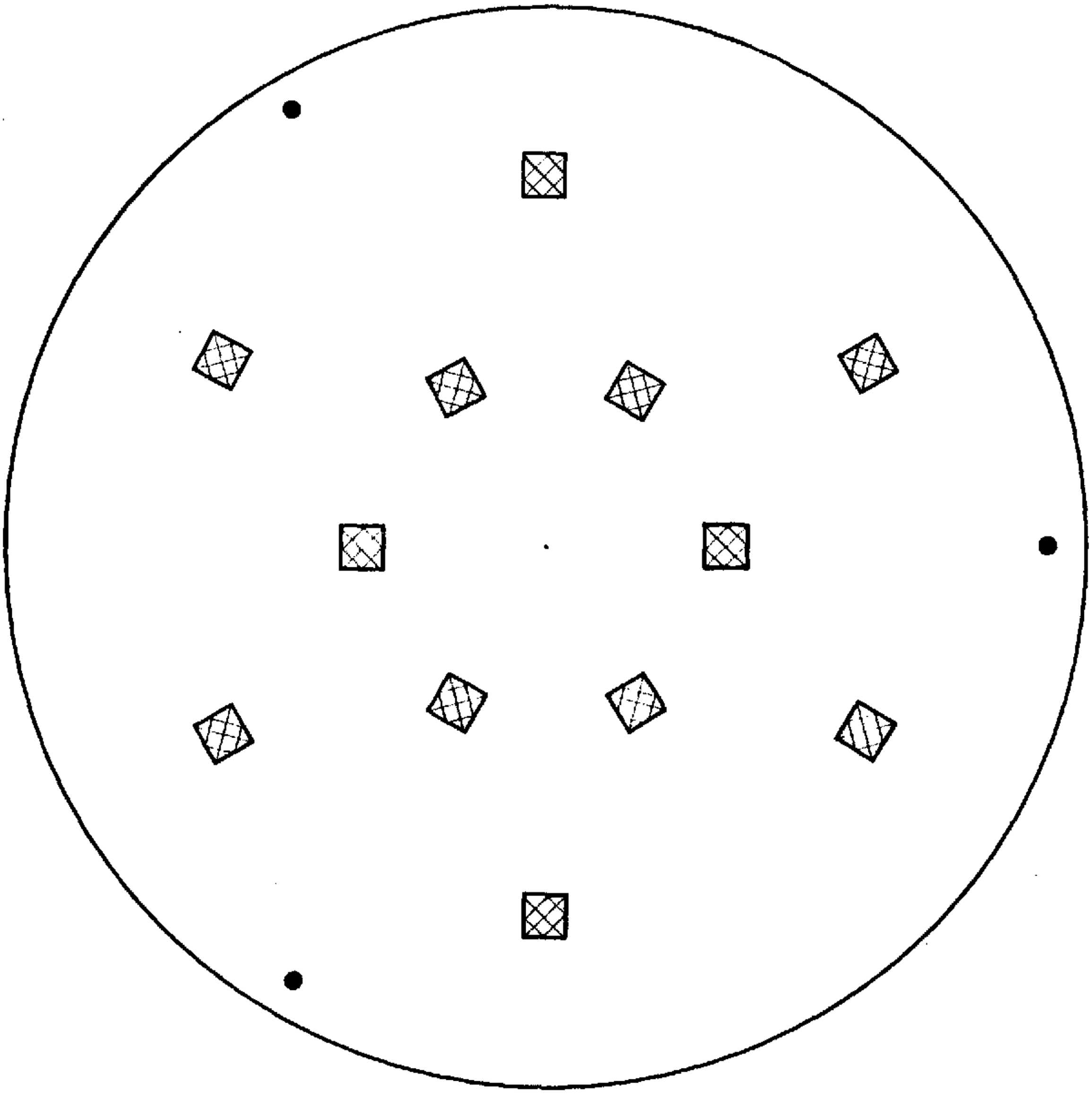


Figure III-3 Array of 12 Heat Patches

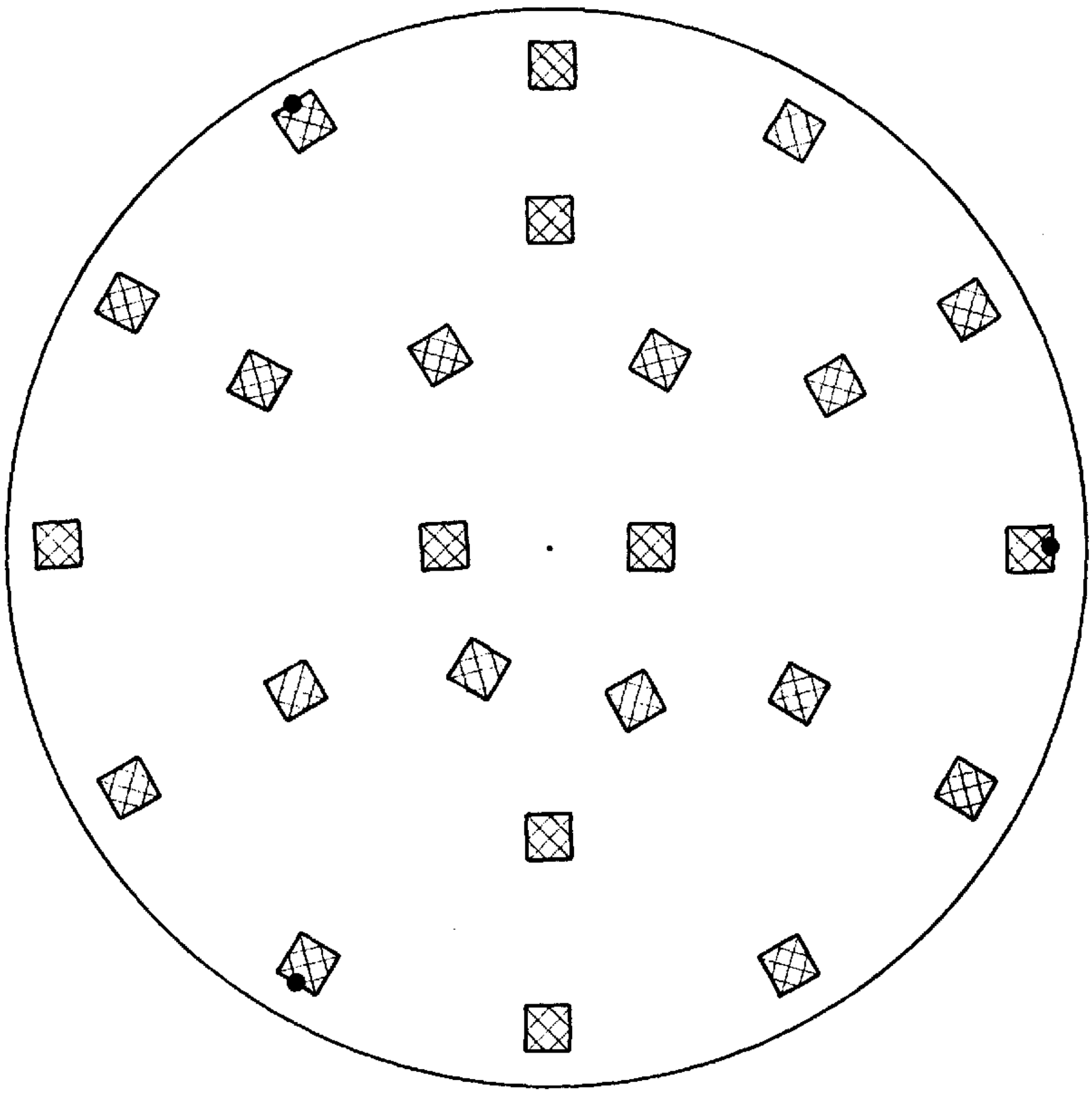
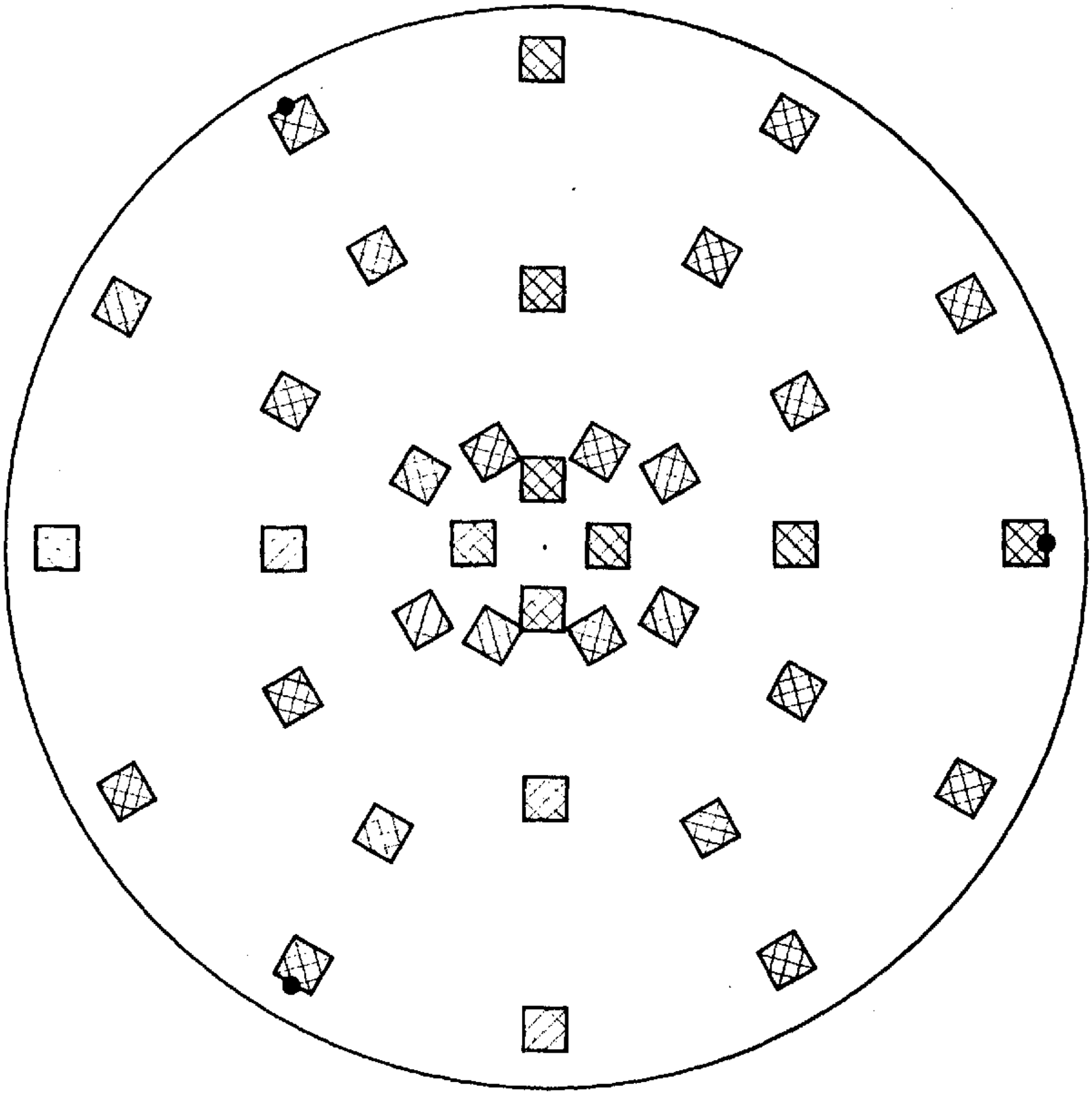


Figure III-4 Array of 24 Heat Patches



● - Support Points

Figure III-5 Array of 36 Heat Patches

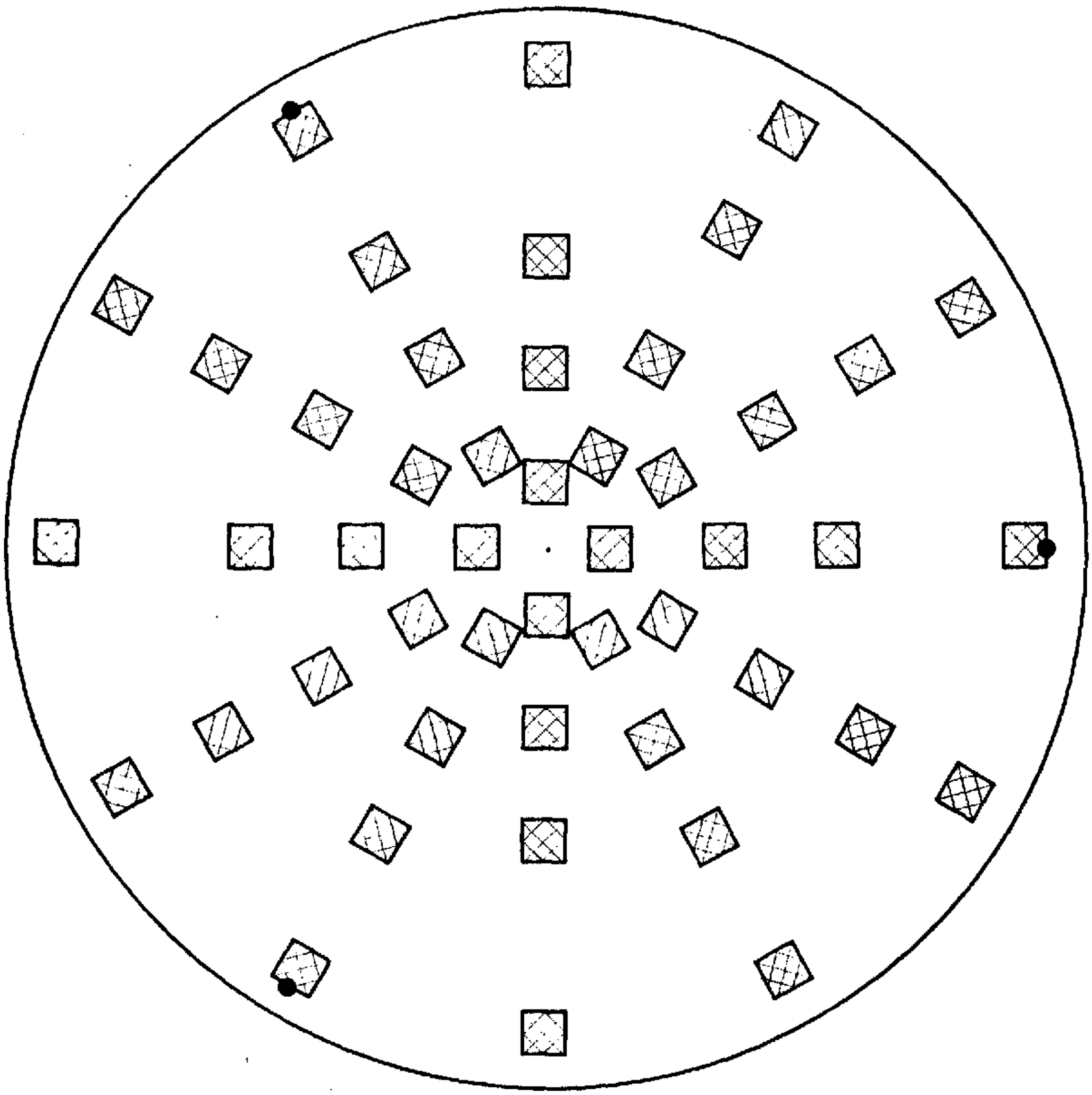


Figure III-6 Array of 48 Heat Patches

IV. PRINCIPLE FEASIBILITY RESULTS

Introduction

The influence matrix corresponding to the mirror described in Table IV-1 was computed using the Response program outlined in Chapter II and Appendix A. The Control program described in Chapter III and Appendix B was used to compute the heat input pattern, unsuppressed surface error, and suppressed (controlled) surface error for a variety of surface errors which are described in the following section. For each error, patterns of 6, 12, 24, 36, and 48 symmetrically located heaters (see Figures III-2 through III-6) were used. The tabulation of results is given in Table IV-2 and discussed in a later section. In each case the error corresponds to the r.m.s. deviation of 180 symmetrically located sample points, (see Figure II-4), about the best-fit-sphere to the points that passes through the supports.

Uncompensated Surface Errors

Five typical surface errors, which were suggested by M.S.F.C., were used for feasibility studies. These are shown in Figures IV-1 through IV-5. The descriptive names of the disturbances are:

(A) High in the middle

(B) Warp

(C) High ring

(D) High spot near support

(E) High spot between supports

Error-Suppression Results

The results obtained by using the control program to suppress the five errors are shown in Table IV-2. A set of 180 unweighted sample

Table IV-1 Dimensions and Properties of NASA Test Mirror

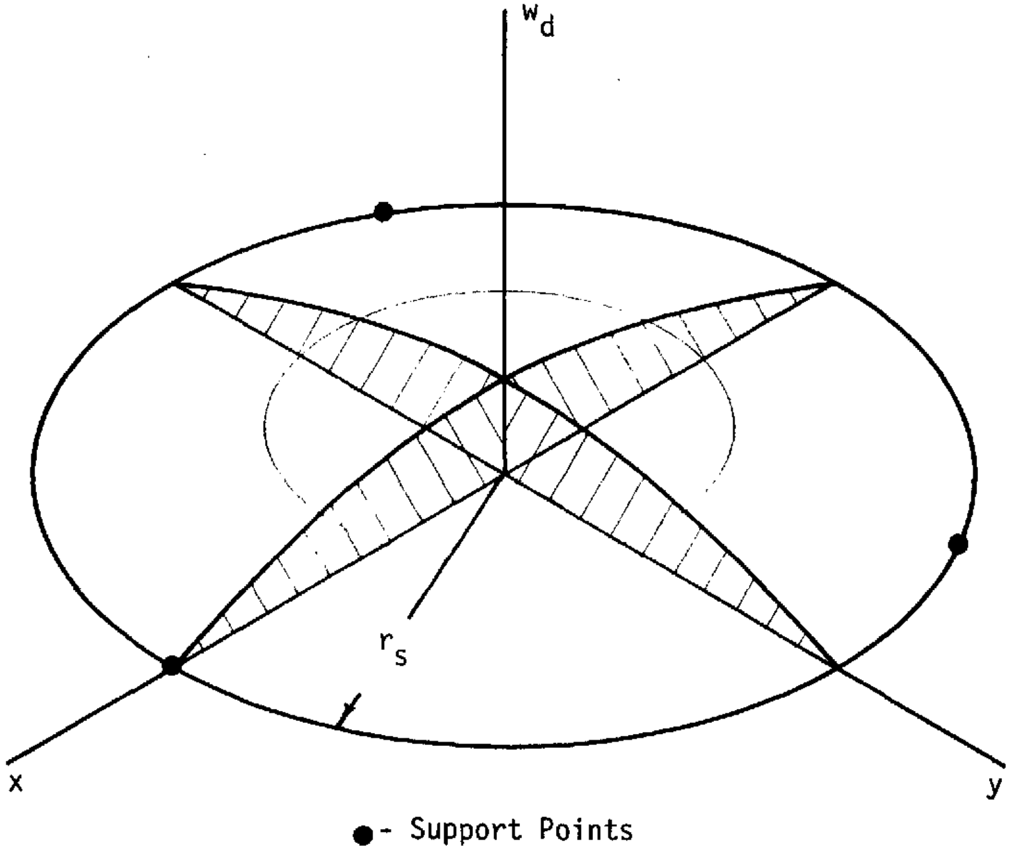
Dimensions:

Inner Radius	0.0
Outer Radius	10.0 in.
Thickness	0.667 in.
F-number	2.0

* Properties:

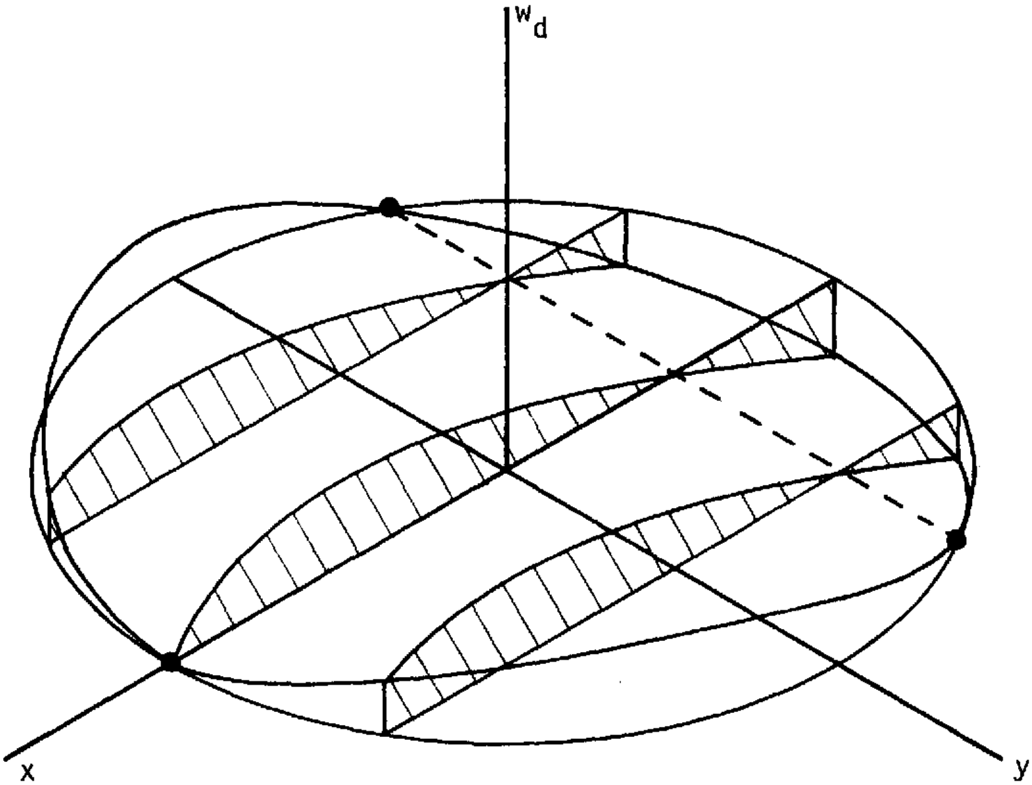
Specific Heat	0.015 Btu/in^3 - °F
Conductivity	0.05 Btu/hr - in - °F
Emissivity	0.04
Young's Modulus	1.06×10^8 lb/in^2
Poisson's Ratio	0.17
Coefficient of Thermal Expansion	0.311×10^{-6}/°F

* This mirror is homogeneous and uniform so the properties do not depend upon direction and are constant throughout the mirror.



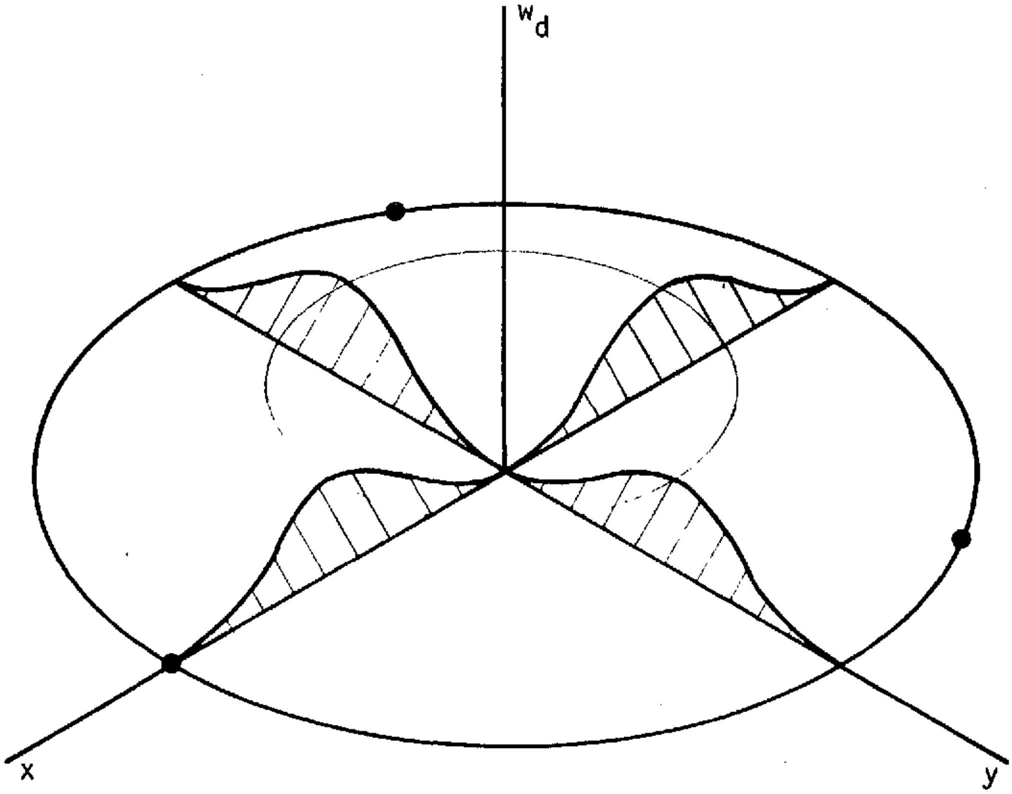
$$w_d(r) = W_o \cos\left(\frac{\pi r}{2r_s}\right)$$

Figure IV-1 Disturbance A - High in the Middle



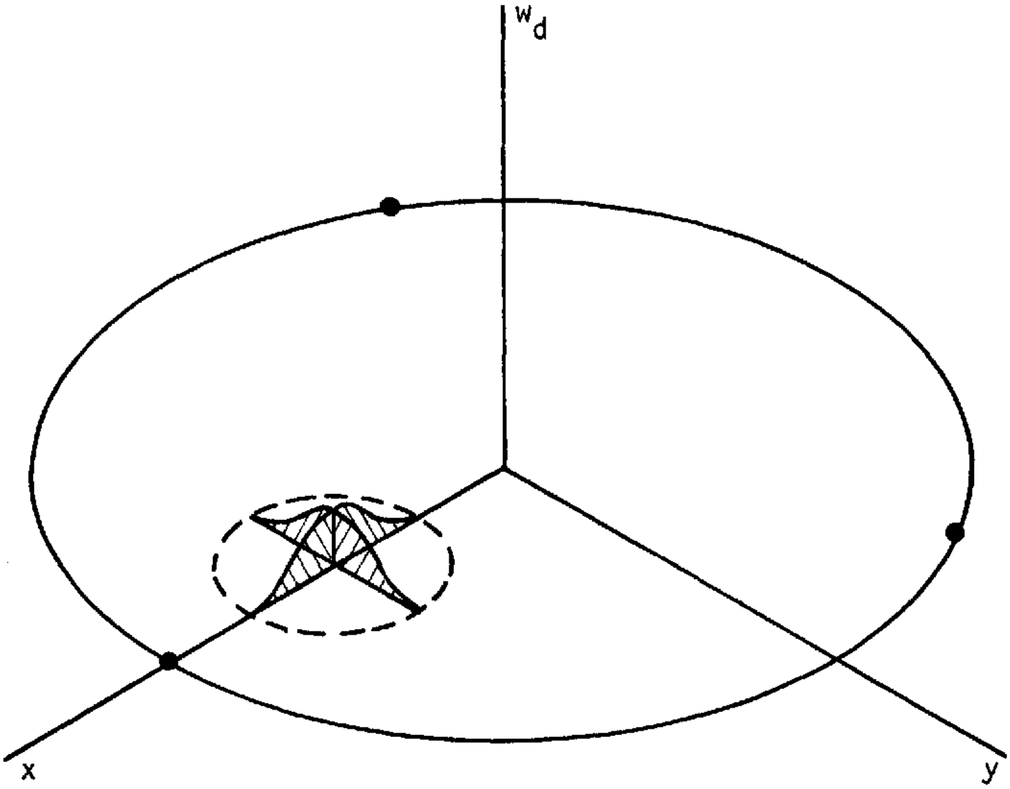
$$w_d(x) = W_o(r_s - x)(x + r_s)$$

Figure IV-2 Disturbance B - Warp



$$w_d(r) = W_o \sin^2(\frac{\pi r}{r_s})$$

Figure IV-3 Disturbance C - High Ring

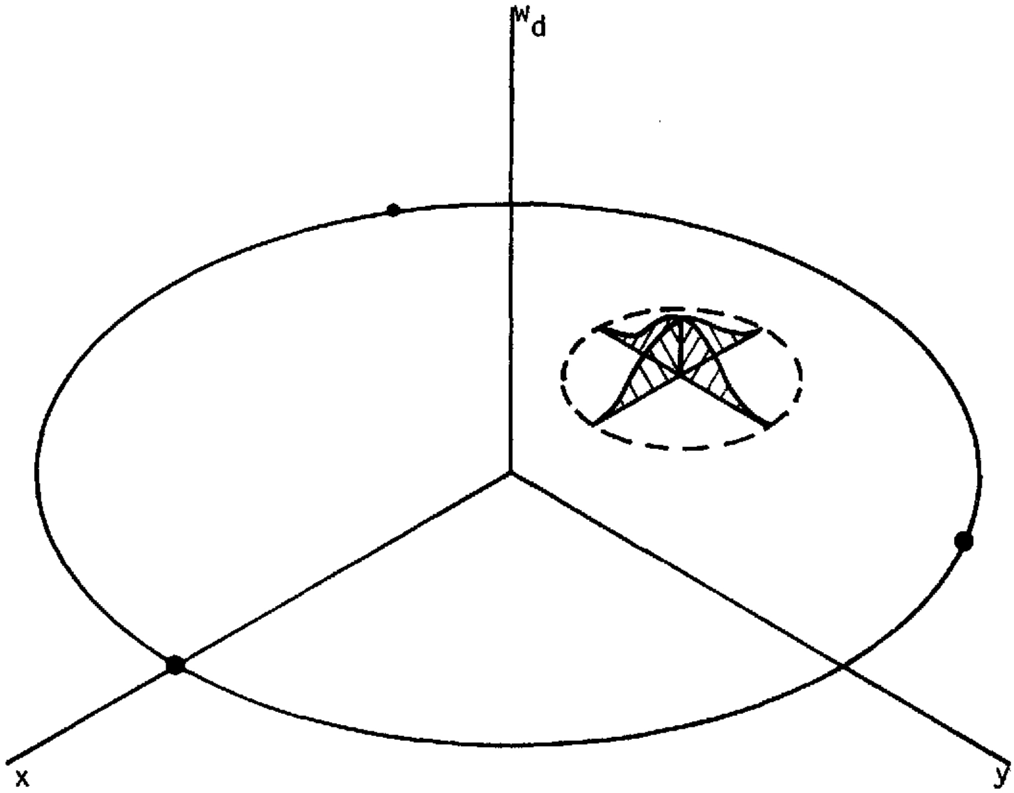


● - Support Points

$$r_1 = [(x - r_s/2)^2 + y^2]^{1/2}$$

$$w_d(x,y) = \begin{cases} 0 & r_1 \geq r_s/4 \\ W_o[1 + \cos(\frac{4\pi r}{r_s})], & r_1 < r_s/4 \end{cases}$$

Figure IV-4 Disturbance D - High Spot Near Support



● - Support Points

$$r_1 = [(x + r_s/2)^2 + y^2]^{1/2}$$

$$w_d(x,y) = \begin{cases} 0 & r_1 \geq r_s/4 \\ W_o[1 + \cos(\frac{4\pi r}{r_s})] & r_1 < r_s/4 \end{cases}$$

Figure IV-5 Disturbance E - High Spot Between Supports

Table IV-2 Error-Suppression Results

Number Heat Patches	Fractional Error with Disturbance				
	A	B	C	D	E
6	.995	.734	.587	.891	.876
12	.992	.551	.576	.842	.846
24	.991	.481	.362	.801	.811
36	.991	.449	.278	.729	.753
48	.991	.434	.268	.562	.580

points was used. The five heat patterns described earlier were used for each error. The error of the compensated mirror is, in each case, normalized by the uncompensated error. Each error is with respect to the best-fit-sphere as described earlier.

A substantial improvement occurs for all errors but the first. The effect of increasing the number of heaters differs from one error to the next. Generally the more localized errors responded well for the larger number of heaters and not particularly well for the small heater patterns.

The less localized errors of warping and high ring were relatively correctable with a small number of heaters. The flattened mirror (A) did not show any appreciable degree of correctability with any of the heat patch distributions. The error reductions as a function of the number of heat patches for the disturbances are presented in Figure IV-6.

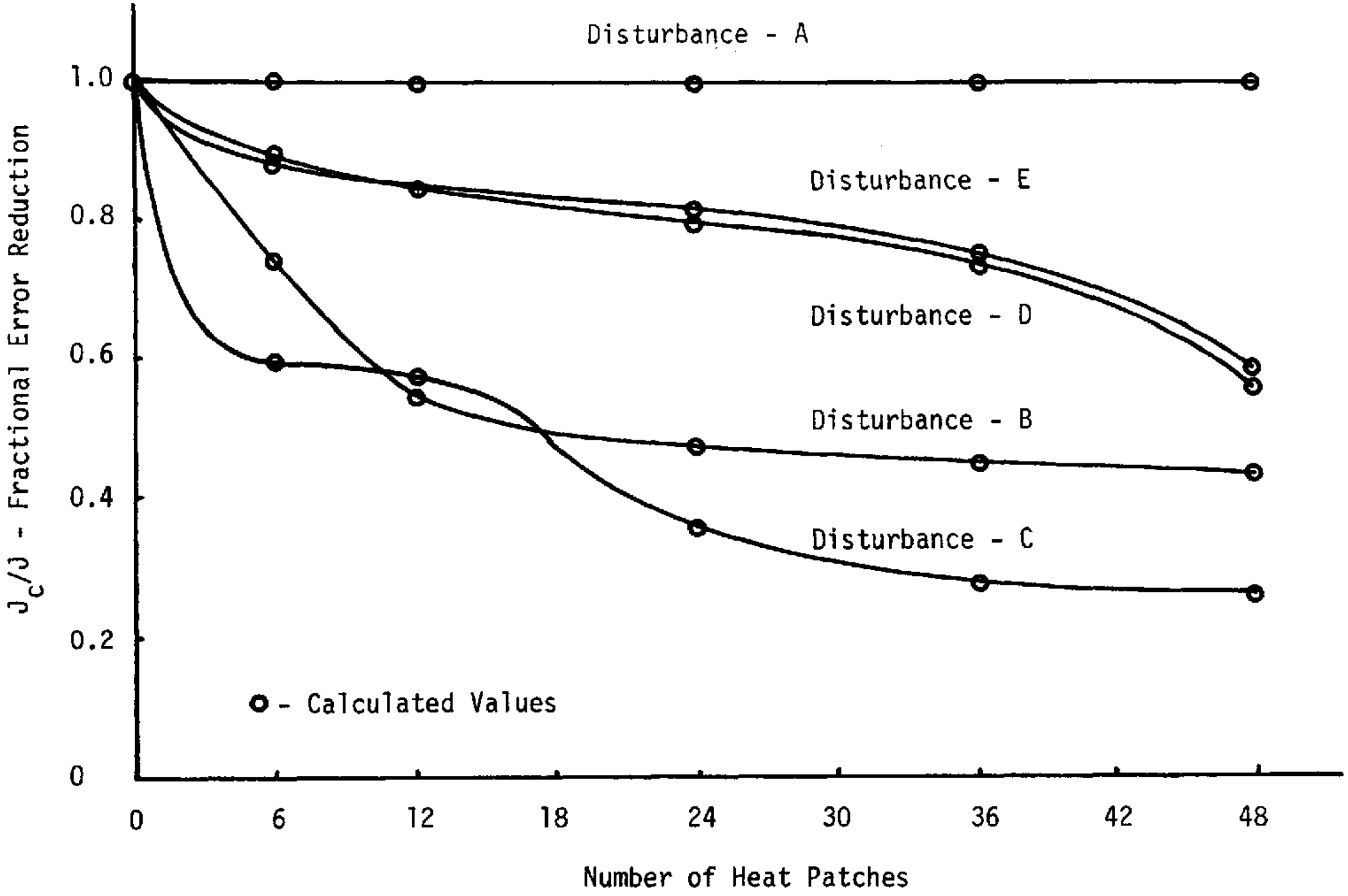


Figure IV-6 Fractional Error Reduction

V. REFERENCES

1. Hill, J.L. and Youngblood, J.N., "Interim Report - Active Control of Primary Mirror of an Orbiting Telescope with Thermal Excitation", BER Report No. 153-09, University of Alabama, May, 1973.

2. Lekhnitskii, S.G., Theory of Elasticity of an Anistropic Elastic Body, Hodden-Day, 1963, pp. 20-21.

APPENDIX A

DESCRIPTION OF THERMOELASTIC RESPONSE PROGRAM

Purpose

This program computes the thermoelastic deflection of a set of nodes specified in an axisymmetric body. Heat is inputed on one surface of the body and radiates at the other.

Options

This program provides the following options:

(1) The temperature and deflection of the node set may be computed at any time.

(2) The steady-state temperature and deflection may be computed.

(3) The influence matrix of surface deflection for a designated heat input pattern may be computed.

Input Parameter Definition

Parameter	Definition
NUMPAT	The total number data cases to be run.
IM	Number of node locations in the radial direction. (Cannot exceed 15.)
JM	Number of node locations in the longitudinal direction. (Cannot exceed 15.)
CF	Radiation coefficient of the front surface.
TO	Temperature of radiating reference medium.
DELT	Integration time.
NTS	Number of integration steps.
IPRINT	Number of integration steps between printings.

Parameter	Definition
TI	Initial mirror temperature.
NM	Highest harmonic (angular) component.
IS	Location of radial node of support ring.
KM	Number of angular divisions. Must not exceed 12.
S2	Angular position of second support (first one is at 0).
S3	Angular position of third support.
INFLU	Switch to choose simulation of response or influence matrix computation. If INFLU = 0 Program calculates the response to heating one patch. If INFLU > 0 Program calculates the thermoelastic influence coefficient matrix. This matrix is written in a data file through I-O unit 4.
IP	Radial node of single patch input.
KP	Angular position of single patch input.
PA	Patch angle for single patch.

Parameter	Definition
PH	Heat input rate for single patch. (Negative is input; positive is output.)
PAN(I)	Patch angle for i-th radial node in full pattern generation.
DO	Outside diameter of mirror.
DI	Inside diameter of mirror.
H	Thickness of mirror.
FNO	F-number of the mirror= Focal Length/Diameter of Mirror.
E(1,M)	E_r of layer M (M=Bottom, Core, Front)
E(2,M)	E_θ of layer M. (See equations II-1)
E(3,M)	E_z of layer M.
G(1,M)	$G_{r\theta}$ of layer M.
G(2,M)	G_{rz} of layer M.
G(3,M)	$G_{\theta z}$ of layer M.
V(1,M)	$\nu_{\theta r}$ of layer M.
V(2,M)	ν_{zr} of layer M.
V(3,M)	$\nu_{z\theta}$ of layer M.
ALF(1,M)	α_r of layer M.
ALF(2,M)	α_θ of layer M.
ALF(3,M)	α_z of layer M.
CK(1,M)	k_r of layer M.
CK(2,M)	k_θ of layer M.
CK(3,M)	k_z of layer M.
CP(M)	Specific thermal capacity of layer M.

Input Data Card Listing

Card No.	Parameter	Data Field	Format
1	NUMPAT	1-5	I5
2	IM	1-5	I5
	JM	6-10	I5
	CF	11-20	F10.5
	TO	21-30	F10.5
	DELT	31-40	F10.5
	NTS	41-45	I5
	IPRINT	46-50	I5
	TI	51-60	F10.5
3	NM	1-5	I5
	IS	6-10	I5
	KM	11-15	I5
	S2	16-20	I5
	S3	21-25	I5
	INFLU	26-30	I5
4	IP	1-5	I5
	KP	6-10	I5
	PA	11-20	F10
	PH	21-30	F10
4	PAN(1)...PAN(7)	1-70	7F10.5
5	PAN(IM-7)...PAN(IM-1)	1-70	7F10.5
6	DO	1-10	F10.5
	DI	11-20	F10.5
	H	21-30	F10.5
	FNO	31-40	F10.5

The following cards contain the material properties for each layer. There should be three sets of these two cards.

7	E(1,M)	1-8	F8.5
	E(2,M)	9-16	F8.5
	E(3,M)	17-24	F8.5
	G(1,M)	25-32	F8.5
	G(2,M)	33-40	F8.5
	G(3,M)	41-48	F8.5
	V(1,M)	49-56	F8.5
	V(2,M)	57-64	F8.5
	V(3,M)	65-72	F8.5
8	ALF(1,M)	1-8	F8.5
	ALF(2,M)	9-16	F8.5
	ALF(3,M)	17-24	F8.5
	CK(1,M)	25-32	F8.5
	CK(2,M)	33-40	F8.5
	CK(3,M)	41-48	F8.5
	CP(M)	49-56	F8.5

Card 2 through the end constitute one data set. There should be NUMPAT data sets.

Output of Program

A. Option INFLU = 0

1. Repeated input data.
2. Table of temperatures and deflection of the node (I=IM, J=JM) for each node for a check of convergence.
3. Table of displacements and temperatures of the nodes on the front surface before the support constraints are added.
4. Table of displacements of the nodes on the front surface after supports are added.

B. Option INFLU = 1

1. Repeated input data.
2. Patch angles of each patch ring.
3. The displacements of the nodal points on the front surface of the mirror corresponding to each patch location are read into a file.

Program Listing

A listing of the program and its subroutines appears on the following pages.

```
C      PROGRAM TO DETERMINE TEMPERATURE DISTRIBUTION AND
C      THERMAL DISTORTION OF A MIRROR
C
C      PARAMETERS
C      CP=SPECIFIC HEAT
C      CK=THERMAL CONDUCTIVITY
C      CF=RADIATION FACTOR FOR FRONT FACE
C      TO=REFERENCE TEMPERATURE
C      I,J=ROW AND COL INDICES OF GRID PT
C      R(I,J),Z(I,J)=COORDINATES OF GRID POINT
C      IM,JM=MAX VALUES OF I AND J
C      NC=NUMBER OF THE HARMONIC COMPONENT
C      TC(I)=TEMPERATURE COMPONENT
C      DELT=INTEGRATION TIME STEP
C      NTS=NUMBER OF TIME INTERVALS
C      ALF=LINEAR COEFFICIENT OF THERMAL EXPANSION
C      E=YOUNGS MODULUS OF ELASTICITY
C      V= POISSONS RATIO
C      DO=OUTSIDE DIAMETER OF MIRROR
C      DI=INSIDE DIAMETER OF MIRROR
C      H=THICKNESS OF MIRROR
C      FNO=F-NUMBER OF THE MIRROR (FOCAL LENGTH/DO)
       DIMENSION U(15,15)
       DIMENSION D(75),R(15,5),Z(15,5),C(75,75,2),QR(15),QB(15,14),
      1TC(75),X(75),A(75,75),Q(75),Q1(75),P(14),F(15,15)
       DIMENSION WF(15,12),WB(12),TT(15,12)
      1,TR(15,12)
       DIMENSION HEAT(100)
       DIMENSION PAN(14)
       DIMENSION WBR(12),WFR(15,12)
       COMMON/STIF1/PD,PS,PL,A
       COMMON/STIF2/KL,KD,KS
       COMMON R,Z
       DIMENSION CN(3,3,3),G(3,3),ALF(3,3),CK(3,3),JL(2),CP(3)
       COMMON/DAT/CN,G,ALF,CK,JL,CP
       INTEGER S1,S2,S3
       COMMON /CND/C,D,QR,QB
       DIMENSION KD(15,15,15),KS(15,15,15),PD(15,5,15),PS(15,5,14),
      3PL(15,5,14),KL(15,15,15)
       DIMENSION AA(6750),DD(6750),BB(225)
       EQUIVALENCE (DD(1),KL(1,1,1))
       REAL KD,KS,KL
C
C    LOOP TO INPUT ALL THE PATCHES
C
       READ (5,3100)NUMPAT
 3100  FORMAT (I5)
       DO 72 NU=1,NUMPAT
       WRITE(6,6500)
 6500  FORMAT(1H1)
       READ(5,100)IM,JM,CF,TO,DELT,NTS,IPRINT,TI
  100  FORMAT(2I5,3F10.5,2I5,F10.5)
       WRITE(6,102)IM,JM,CF,TO,DELT,NTS
       READ(5,2600)NM,IS,KM,S2,S3,INFLU
 2600  FORMAT(6I5)
       NMP=NM+1
```

```
      WRITE(6,2800)TI
 2800 FORMAT(27H INITIAL TEMPERATURE, TI = ,F10.5/////)
      WRITE(6,2601)NMP,KM,IS,S2,S3,INFLU
 2601 FORMAT(33H NUMBER OF HARMONIC COMPONENTS = ,I5/
     137H NUMBER OF ANGULAR PATCH POSITIONS = ,I5/
     231H RADIAL NODE OF SUPPORT RING = ,I5/
     348H ANGULAR POSITIONS OF SUPPORTS ARE,K =  1,  K = ,I2,2X,
     410H AND, K = ,I2/
     5  9H INFLU = ,I2)
      IX=IS
      N1=3
      MAT=IM
      FKM=KM
      NTP=NTS+1
      IM1=IM-1
      M=IM*JM
      FJM1=JM-1
      FIM1=IM1
      IF(INFLU.GT.0)GO TO 2500
      READ(5,2602)IP,KP,PA,PH
 2602 FORMAT(2I5,2F10.5)
      WRITE(6,2603)IP,KP,PA,PH
 2603 FORMAT (51H THERMOELASTIC RESPONSE OF THE MIRROR IS CALCULATED/
     124H RADIAL NODE OF PATCH = ,I5/
     229H ANGULAR POSITION OF PATCH = ,I5/
     315H PATCH ANGLE = ,F10.5/
     414H PATCH HEAT = ,F10.5)
      IPLO=IP+1
      GO TO 2501
 2500 READ(5,2604)(PAN(I),I=1,IM1)
      READ(5,3500) IPLO,IPHI
 3500 FORMAT(2I5)
 2604 FORMAT(7F10.5)
      WRITE(6,2605)(I,PAN(I),I=1,IM1)
 2605 FORMAT(72H THERMOELASTIC INFLUENCE COEFFICIENTS ARE CALCULATED AND
     1 WRITTEN ON FILE/
     24X,1HI,5X,11HPATCH ANGLE/
     3(I5,5X,F10.5))
      WRITE(6,3501) IPLO,IPHI
 3501 FORMAT(8H IPLO = ,I2,5X,8H IPHI = ,I2)
      IF(IPLO.EQ.1) GO TO 2501
      KSTOP=(IPLO-1)*12
      DO 3503 KRD=1,KSTOP
 3503 READ(4,1600)((WFR(I,K),I=1,IM),K=1,KM)
 2501 CONTINUE
      PI=3.1415927
C
C     GRID GENERATION FOR SPHERICAL MIRROR
C
      READ(5,112)DO,DI,H,FNO
      WRITE (6,113) DO,DI,H,FNO
      IF(FNO.GT.99)    GO TO 201
      RF=2.*DO*FNO
      RB=RF+H
      DR=H/FJM1
      SI=.5*DI/RF
      CI=SQRT(1.-SI*SI)
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```
      SO=.5*DO/RF
      CO=SQRT(1.-SO*SO)
      THETI=ATAN(SI/CI)
      THETO=ATAN(SO/CO)
      DTHET=(THETO-THETI)/FIM1
      EM=THETI
      DO 39 I=1,IM
      RR=RB
      DO 38 J=1,JM
      R(I,J)=RR*SIN(EM)
      Z(I,J)=RB-RR*COS(EM)
   38 RR=RR-DR
   39 EM=EM+DTHET
      GO TO 202
  201 DH=.5*(DO-DI)/FIM1
      DZ=H/FJM1
      DO 200 I=1,IM
      FI=I-1
      DO 200 J=1,JM
      FJ=J-1
      R(I,J)=0.5*DI+FI*DH
  200 Z(I,J)=FJ*DZ
 202  CONTINUE
      CALL DATA(JM)
C
C     CALL SUBROUTINE TO CALCULATE C,D,KD,KS,PD,PS,PL
C
      CALL CAND(IM,JM,M,CF)
      SCP=CP(1)+CP(2)+CP(3)
C
COMPONENTS OF PATCH HEAT
C
      IF(INFLU.LE.0)GO TO 2502
      IP=IPLO
 2505 PA=PAN(IP)
      IP1=IP+1
      AP=PA*3.14159265/360.*(R(IP1,1)**2-R(IP,1)**2)
      PH=-1./AP
 2502 CONTINUE
      DO 2000 N=1,NM
      FN=N
      HEATO=PH*PA/360.
 2000 HEAT(N)=(2.*PH*SIN(FN*PA*PI/360.))/(FN*PI)
C
C  INITIALIZE WF AND WB
C
      DO 2001 K=1,KM
      WB(K)=0.0
      DO 2001 I=1,IM
      TT(I,K)=0.0
 2001 WF(I,K)=0.0
C
C    START MODE LOOP
C
      IF(INFLU.GT.0) GO TO 6000
      WRITE(6,500)
  500 FORMAT(/////31H CONVERGENCE OF MODAL RESPONSES//
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     11X,4HMODE,11X,10HHEAT INPUT,4X,1HI,4X,1HJ,10X,10HDEFLECTION,
     29X,11HTEMPERATURE)
 6000 DO 2002 NT=1,NMP
      N1=3
      NC=NT-1
      IF(NC.EQ.0)N1=2
      JM3=N1*JM
      LCV=JM3
      IJM=IM*JM3
      CALL STIFF(IM,JM,NC,CF)
C
C       REARRANGE KD AND KS AND FORM KL FOR POTTER
C
      DO 50 I=1,IM
      DO 50 L1=1,JM3
      DO 50 L2=1,JM3
   50 KL(L1,L2,I)=KD(L1,L2,I)
      DO 51 I=1,IM
      DO 51 L1=1,JM3
      DO 51 L2=1,JM3
   51 KD(I,L1,L2)=KL(L1,L2,I)
      DO 52 I=1,IM
      DO 52 L1=1,JM3
      DO 52 L2=1,JM3
   52 KL(L1,L2,I)=KS(L1,L2,I)
      DO 53 I=1,IM
      DO 53 L1=1,JM3
      DO 53 L2=1,JM3
  53  KS(I,L1,L2)=KL(L1,L2,I)

      DO 9500 L1=1,15
      DO 9500 L2=1,15
      DO 9500 L3=1,15
 9500 KL(L1,L2,L3)=0.0
      DO 54 I=2,IM
      DO 54 L1=1,JM3
      DO 54 L2=1,JM3
 54   KL(I,L1,L2)=KS(I-1,L2,L1)
C       INITIAL CONDITIONS ON TEMPERATURE
C
      IF(NC.NE.0) GO TO 131
      DO 37 I=1,M
   37 TC(I)=TI
      GO TO 132
  131 DO 133 I=1,M
  133 TC(I)=0.0
  132 CONTINUE
C
C     INITIALIZE DISPLACEMENTS
C
      DO 24 L1=1,IM
      DO 24 L2=1,JM3
   24 U(L1,L2)=0.0
C
C     RADIATION FROM FRONT
C
      FC=NC
```

```
      DO 20 L1=1,M
      X(L1)=0.0
 20   Q1(L1)=0.0
      IF(NC.NE.0) GO TO 22
      DO 21 L1=1,IM
      LR=L1*JM
  21    Q1(LR)=Q1(LR)+   TO*QR(L1)
 22   CONTINUE
C
C     SET UP AND INVERT EFFECTIVE CONDUCTIVE MATRIX IN A
C
      DO 2 L1=1,M
      DO 2 L2=1,M
      A(L1,L2)=C(L1,L2,1)+FC*FC*C(L1,L2,2)
      IF(L1.EQ.L2)A(L1,L2)=A(L1,L2)+2.0*D(L1)/DELT
    2 CONTINUE
      CALL INVERT(A,M,75)
C
C     INITIALIZE SOLUTION AND HEAT INPUT
C
      DO 99 I=1,IM1
   99 P(I)=0.0
C
C     STEP BY STEP SOLUTION
C
      IPR=0
      IF(SCP.GT.0.0001) GO TO 73
      NTP=2
      IPRINT=1
 73   DO 6 II=1,NTP
      DO 45 K1=1,M
 45   X(K1)=0.0
      FI=II-1
       TIME=FI*DELT
      IF(II.EQ.1)GO TO 40
      IPR=IPR+1
C
C     HEAT INPUT ON BACK
C
      IF(NC .NE. 0) P(IP)=HEAT(NC)
      IF(NC .EQ. 0) P(IP)= HEATO
      DO 28 L=2,IM1
      IJ=(L-1)*JM+1
   28 Q1(IJ)=QB(L,L-1)*P(L-1)+QB(L,L)*P(L)
      Q1(1)=QB(1,1)*P(1)
      I2=M+1-JM
      Q1(I2)=QB(IM,IM1)*P(IM1)
C
C     SET UP THE COMBINE HEAT INPUT VECTOR
C
      DO 3 L1=1,M
    3 Q(L1)=2.0*D(L1)*TC(L1)/DELT +Q1(L1)
C
C     CALCULATE THE NEW TEMPERATURE
C
 5002 DO 10 L1=1,M
      DO 10 L2=1,M
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```
10    X(L1)=X(L1)+A(L1,L2)*Q(L2)
      IF(SCP.LT.0.0001) GO TO 70
      DO 4 L1=1,M
    4 TC(L1)=-TC(L1)+2.*X(L1)
      IF(IPR.NE.IPRINT)GO TO 6
      IPR=0
 5001 CONTINUE
      IF(SCP.GT.0.0001) GO TO 5000
 70   DO 62 L1=1,M
   62 TC(L1)=X(L1)
      IF(NT.EQ.1)GO TO 8600
      IF(DI.GT.0.0001)GO TO 8600
      DO 8601 J=1,JM
 8601 TC(J)=0.0
 8600 CONTINUE
C
C     SET UP THE THERMAL FORCE VECTOR
C
 5000 IF(NC.NE.0) GO TO 71
      DO 55 L1=1,M
   55 TC(L1)=TC(L1)-TI
   71 DO 41 L1=1,IM
      DO 41 L2=1,JM3
   41 F(L1,L2)=0.0
      DO 43 I1=1,IM
      DO 43 L2=1,JM
      DO 43 J=1,JM
      DO 42 K=1,N1
      L1=N1*(J-1)+K
      L3=JM*(I1-1)+L2
      L4=L3+JM
      L5=L3-JM
      F(I1,L1)=-PD(L1,L2,I1)*TC(L3)+F(I1,L1)
      IF(I1.NE.IM)F(I1,L1)=F(I1,L1)-PS(L1,L2,I1)*TC(L4)
      IF(I1.EQ.1)GO TO 42
      F(I1,L1)=F(I1,L1)-PL(L1,L2,I1-1)*TC(L5)
   42 CONTINUE
  43  CONTINUE
C
C     IMPOSE BOUNDARY CONDITIONS
C
      N=NC
      IF(N.NE.0)GO TO 49
      DO 44 L1=1,JM3
      IF(IX.NE.1)KS(IX-1,L1,2)=0.
      IF(IX.NE.IM)KL(IX+1,L1,2)=0.
      KD(IX,L1,2)=0.
      KS(IX,2,L1)=0.0
      IF(IX.NE.1)KL(IX,2,L1)=0.0
   44 KD(IX,2,L1)=0.0
      KD(IX,2,2)=1.0
      F(IX,2)=0.0
      GO TO 48
 49   IF(N.NE.1)GO TO 48
      DO 46 L1 =1,JM3
      KL(IX,1,L1)=0.0
      KL(IX,3,L1)=0.0
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```
      KL(IX+1,L1,1)=0.0
      KL(IX+1,L1,3)=0.0
      KS(IX,1,L1)=0.0
      KS(IX,3,L1)=0.0
      KD(IX,1,L1)=0.0
      KD(IX,3,L1)=0.0
      KD(IX,L1,1)=0.0
      KD(IX,L1,3)=0.0
      IF(IX.NE.1)KS(IX-1,L1,1)=0.0
 46   IF(IX.NE.1)KS(IX-1,L1,3)=0.0
      KD(IX,1,1)=1.0
      KD(IX,3,3)=1.0
      F(IX,1)=0.0
      F(IX,3)=0.0
   48 CONTINUE
      IF(N.EQ.0)GO TO 8500
      IF(DI.GT.0.0001)GO TO 8500
      DO 8502 L1=1,JM3
      DO 8501 L2=1,JM3
      KD(1,L1,L2)=0.0
      KS(1,L1,L2)=0.0
 8501 KL(2,L1,L2)=0.0
      KD(1,L1,L1)=1.0
 8502 F(1,L1)=0.0
 8500 CONTINUE
      DO 8000 L1=1,6750
 8000 AA(L1)=0.0
      DO 8001 L1=1,225
 8001 BB(L1)=0.0
      DO 8002 L1=1,IM
      DO 8002 L2=1,JM3
      DO 8002 L3=L2,JM3
      L=L3+2 *(L2-1 )*JM3-L2+1 +(L1-1 )*2 *JM3*JM3
 8002 AA(L)=KD(L1,L2,L3)
      DO 8003 L1=1,IM1
      DO 8003 L2=1,JM3
      DO 8003 L3=1,JM3
      L=L3+2 *(L2-1 )*JM3-L2+1 +(L1-1 )*2*JM3*JM3+JM3
 8003 AA(L)=KS(L1,L2,L3)
      DO 8004 L1=1,IM
      DO 8004 L2=1,JM3
      L=L2+(L1-1)*JM3
 8004 BB(L)=F(L1,L2)
      NEQ=JM3*IM
      MB=JM3+JM3
      DO 8050 L3=1,NEQ
      DO 8050 L4=1,MB
      L1=L3+(L4-1)*NEQ
      L2=L4+(L3-1)*MB
 8050 DD(L1)=AA(L2)
      MOS=MB*NEQ
      DO 8060 L1=1,MOS
 8060 AA(L1)=DD(L1)
C
C     SOLVE THE STIFFNESS EQUATIONS
C
      CALL TRIA (NEQ,MB,AA)
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```
      CALL BACKS (NEQ,MB,AA,BB)
      DO 8005 L1=1,IM
      DO 8005 L2=1,JM3
      L=L2+(L1-1)*JM3
 8005 U(L1,L2)=BB(L)
   40 CONTINUE
    6 CONTINUE
C
COMPUTE WB AND WF
C
      IF(NC.NE.0)GO TO 4000
      DO 4001 L1=1,M
 4001 TC(L1)=TC(L1)+TI
 4000 CONTINUE
      DO 2003 K=1,KM
      FN=NC
      FK=K
      WB(K)=WB(K)+U(IS,N1)*COS(FN*(FK-1.)*PI*2./FKM)
      DO 2003 I=1,IM
      IT=I*JM
      TT(I,K)=TT(I,K)+TC(IT)*COS(FN*(FK-1.)*PI*2./FKM)
 2003 WF(I,K)=WF(I,K)+U(I  ,JM3)*COS(FN*(FK-1.)*PI*2./FKM)
C
C     OUTPUT RESULTS OF A MODE
C
      IF (INFLU.GT.0)GO TO 2002
      I=IM
      J=JM
      JJ=J*N1
      JJJ=JM*(I-1)+J
      WRITE(6,501) NC,P(IP),I,J,U(I,JJ),TC(JJJ)
  501 FORMAT(I5,E20.8,2I5,2E20.8)
 2702 CONTINUE
 2002 CONTINUE
C
C     ROTATION OF OUTPUT FOR SUPPORTS
C
 3000 CONTINUE
      S1=1
      IF(INFLU.LE.0)GO TO 2503
      KP=1
 2503 CONTINUE
      FKM=KM
      FS2=S2
      FS3=S3
      DTHET=2.*PI/FKM
      DTHD=360./FKM
      KR=KP-1
      DO 1001 K=1,KM
      KR=KR+1
      IF (KR.EQ. KM+1) KR=1
      DO 1002 I=1,IM
      TR(I,KR)=TT(I,K)
 1002 WFR(I,KR)=WF(I,K)
 1001 WBR(KR)=WB(K)
 1000 CONTINUE
      W1=WBR(S1)
```

```
      W2=WBR(S2)
      W3=WBR(S3)
      RS=R(IS,1)
      THET2=(FS2-1.)*DTHET
      THET3=(FS3-1.)*DTHET
      X1=RS
      X2=RS*COS(THET2)
      X3=RS*COS(THET3)
      Y2=RS*SIN(THET2)
      Y3=RS*SIN(THET3)
      DET=Y3*(X2-X1)+Y2*(X1-X3)
      IF(DET.GT.0.0001)GO TO 1003
      WRITE(6,1510)
 1510 FORMAT(23H SUPPORTS LOCATED WRONG)
      GO TO 72
 1003 WO=((X2*Y3-X3*Y2)*W1-X1*Y3*W2+X1*Y2*W3)/DET
      THX=((X3-X2)*W1+(X1-X3)*W2+(X2-X1)*W3)/DET
      THY=((Y3-Y2)*W1-Y3*W2+Y2*W3)/DET
      DO 1004 K=1,KM
      FK=K
      DO 1005 I=1,IM
      RI=R(I,JM)
      TK=(FK-1.0)*DTHET
      XIK=RI*COS(TK)
      YIK=RI*SIN(TK)
 1005 WFR(I,K)=WFR(I,K)-WO-YIK*THX+XIK*THY
      XIS=RS*COS(TK)
      YIS=RS*SIN(TK)
 1004 WBR(K)=WBR(K)-WO-YIS*THX+XIS*THY
      IF(INFLU.GT.0)GO TO 3002
      WRITE (6,1501)
      DO 1506 K=1,KM
      WRITE(6,1505)
      FK=K-1
      THD=FK*DTHD
      DO 1506 I=1,IM
 1506 WRITE(6,1502)I,K,R(I,JM),THD,WFR(I,K),TR(I,K)
 3002 CONTINUE
      IF(INFLU.LE.0)GO TO 72
 5099 FORMAT(6H IP = ,I5,5X,5HKP = ,I5)
      WRITE(4,1600)((WFR(I,K),I=1,IM),K=1,KM)
 1600 FORMAT(6E13.8)
      WRITE(6,5099) IP,KP
 7005 CONTINUE
      IF (INFLU.GT.0) GO TO 3003
      WRITE (6,1503)
      DO 1507 K=1,KM
      FK=K-1
      THD=FK*DTHD
 1507 WRITE(6,1504)K,RS,THD,WBR(K)
 1501 FORMAT (29H Z-DISPLACEMENTS ON THE FRONT //4X,1HI,4X,1HK,10X,
     11HR,14X,5HTHETA,12X,1HW,16X,11HTEMPERATURE)
 1502 FORMAT(2I5,2E15.5,2E20.8)
 1505 FORMAT (//)
 1503 FORMAT(//50H Z-DISPLACEMENTS AT THE SUPPORT RADIUS ON THE BACK//
     14X,1HK,14X,2HRS,13X,5HTHETA,10X,1HW)
 1504 FORMAT(I5,3E20.8)
```

```
3003 CONTINUE
 102 FORMAT(////39H TEMPERATURES AND THERMAL DISPLACEMENTS,///
    119H NO. OF GRID ROWS = ,I5,5X,20H NO. OF GRID COLS. = ,I5/
    2/18H MIRROR PROPERTIES/
    424H RADIATION FACTOR, CF = ,E15.6/29H REFERENCE TEMPERATURE, TO =
    5,E15.6//24H SOLUTION TIMER CONTROLS/32H INTEGRATION TIME STEP, DEL
    6TA = ,E15.6,5X,28HNUMBER OF TIME STEPS, NTS = ,I5//)
 112 FORMAT(4F10.5)
 113 FORMAT(////16H MIRROR GEOMETRY//20H OUTSIDE DIAMETER = ,F10.5/
    119H INSIDE DIAMETER = ,F10.5/13H THICKNESS = ,F10.5/12H F-NUMBER =
    2 ,F10.5///)
      IF(INFLU.LE.0)GO TO 72
      KP=KP+1
      IF(KP.GT.KM)GO TO 2504
      GO TO 2503
2504  IP=IP+1
      IF(IP.LT.IPHI) GO TO2505
72    CONTINUE
      STOP
      END
```

```
      SUBROUTINE CAND(IM,JM,M,CF)
      DIMENSION CN(3,3,3),S(3,3),ALF(3,3),CK(3,3),JL(2),CP(3)
      COMMON/DAT/CN,S,ALF,CK,JL,CP
      COMMON R,Z
      COMMON /CND/C,D,QR,QB
      DIMENSION C(75,75,2),D(75),R(15,5),Z(15,5)
      DIMENSION CT(3,3,2),CQ(5,5,2),GT(3,3,2),DQ(4),A(3,3),F1(5,5,2),
     2F2(5,5,2),G(5,5,2),D1(5),D2(5),GQ(3),QR(15),QB(15,14)
      DIMENSION RZ(3),ZR(3),X(10)
      IM1=IM-1
      JM1=JM-1
      DO 20 L1=1,M
      D(L1)=0.0
      DO 20 L2=1,M
      DO 20 L3=1,2
   20 C(L1,L2,L3)=0.0
      DO 47 L=1,IM
   47 QR(L)=0.0
      DO 60 M1=1,IM
      DO 60 M2=1,IM1
60    QB(M1,M2)=0.0
      DO 25 L1=1,JM
      D1(L1)=0.0
      D2(L1)=0.0
      DO 25 L2=1,JM
      DO 25 L3=1,2
      F1(L1,L2,L3)=0.0
      F2(L1,L2,L3)=0.0
   25 G(L1,L2,L3)=0.0
      DO 24 I=1,IM
      DO 46 L=1,JM
      D2(L)=0.0
      DO 46 LL=1,2
      DO 46 K=1,JM
      G(L,K,LL)=0.0
   46 F2(L,K,LL)=0.0
      DO 21 J=1,JM1
      IF (I.EQ.IM)GO TO 21
      IF(J.LT.JL(1))MT=1
      IF(J.GE.JL(1).AND.J.LT.JL(2))MT=2
      IF(J.GE.JL(2))MT=3
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      R3=R(I+1,J+1)
      Z3=Z(I+1,J+1)
      R4=R(I,J+1)
      Z4=Z(I,J+1)
      R21=R2-R1
      R32=R3-R2
      R41=R4-R1
      R34=R3-R4
      Z21=Z2-Z1
      Z32=Z3-Z2
      Z41=Z4-Z1
```

```
      Z34=Z3-Z4
      AREA=.5*(R41*Z41-R21*Z21-R32*Z32+R34*Z34)-R32*Z21+R34*Z41
      AR=.5*(R41*Z41)*(R1+2.*R41/3.0)-.5*R21*Z21*(R1+2.*R21/3.)-R32*Z21*
     3(R2+.5*R32)-.5*R32*Z32*(R2+2.*R32/3.)+.5*R34*Z34*(R4+2.*R34/3.)+R3
     44*Z41*(R4+.5*R34)
      AZ=.5*R41*Z41*(Z1+Z41/3.)-.5*R21*Z21*(Z1+Z21/3.)-R32*Z 1*(Z1+.5*Z2
     51)-.5*R32*Z32*(Z2+Z32/3.)+.5*R34*Z34*(Z4+Z34/3.)+R34*Z41*(Z1+.5*Z4
     61)
      RC=AR/AREA
      ZC=AZ/AREA
      DO 16 L1=1,5
      DO 16 L2=1,5
      DO 16 L3=1,2
   16 CQ(L1,L2,L3)=0.0
      DO 17 L1=1,4
   17 DQ(L1)=0.0
      DO 15 K=1,4
      IF(K.NE.1)GO TO 1
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      GO TO 4
    1 IF(K.NE.2)GO TO 2
      R1=R(I+1,J)
      Z1=Z(I+1,J)
      R2=R(I+1,J+1)
      Z2=Z(I+1,J+1)
      GO TO 4
    2 IF(K.NE.3)GO TO 3
      R1=R(I+1,J+1)
      Z1=Z(I+1,J+1)
      R2=R(I,J+1)
      Z2=Z(I,J+1)
      GO TO 4
    3 IF(K.NE.4)GO TO 4
      R1=R(I,J+1)
      Z1=Z(I,J+1)
      R2=R(I,J)
      Z2=Z(I,J)
    4 R3=RC
      Z3=ZC
      AT=0.5*(R2*Z3-R3*Z2+R3*Z1-R1*Z3+R1*Z2-R2*Z1)
      A(1,1)=.5*(R2*Z3-R3*Z2)/AT
      A(2,1)=.5*(Z2-Z3)/AT
      A(3,1)=.5*(R3-R2)/AT
      A(1,2)=.5*(R3*Z1-R1*Z3)/AT
      A(2,2)=.5*(Z3-Z1)/AT
      A(3,2)=.5*(R1-R3)/AT
      A(1,3)=.5*(R1*Z2-R2*Z1)/AT
      A(2,3)=.5*(Z1-Z2)/AT
      A(3,3)=.5*(R2-R1)/AT
      DO 5 L1=1,3
      GG(L1)=0.0
      DO 5 L2=1,3
      DO 5 L3=1,2
      GT(L1,L2,L3)=0.0
```

```
 5 CT(L1,L2,L3)=0.0
   IF(ABS(R1-R2).LT.0.000001)GO TO 6
   ALF12=(Z1-Z2)/(R1-R2)
   BET12=(Z2*R1-Z1*R2)/(R1-R2)
   GO TO 7
 6 ALF12=0.0
   BET12=0.0
 7 IF(ABS(R2-R3).LT.0.000001)GO TO 8
   ALF23=(Z2-Z3)/(R2-R3)
   BET23=(Z3*R2-Z2*R3)/(R2-R3)
   GO TO 9
 8 ALF23=0.
   BET23=0.
 9 IF(ABS(R3-R1).LT.0.000001)GO TO 10
   ALF31=(Z3-Z1)/(R3-R1)
   BET31=(Z1*R3-Z3*R1)/(R3-R1)
   GO TO 11
10 ALF31=0.0
   BET31=0.0
11 CONTINUE
   RZ(1)=R1
   RZ(2)=R2
   RZ(3)=R3
   ZR(1)=Z1
   ZR(2)=Z2
   ZR(3)=Z3
   CALL INTGRL(RZ,ZR,X)
   GT(2,2,1)=X(3)*CK(1,MT)
   GT(3,3,1)=X(3)*CK(3,MT)
   GT(1,1,2)=X(1)
   GT(1,2,2)=X(2)
   GT(1,3,2)=X(6)
   GT(2,1,2)=X(2)
   GT(2,2,2)=X(3)
   GT(2,3,2)=X(7)
   GT(3,1,2)=X(6)
   GT(3,2,2)=X(7)
   GT(3,3,2)=X(10)
   DO 50 L1=1,3
   DO 50 L2=1,3
50 GT(L1,L2,2)=GT(L1,L2,2)*CK(2,MT)
   IF(J.NE.JM1)GO TO 51
   IF(K.NE.3)GO TO 51
   SA=SQRT(1.+ALF12**2)
   N=1
   GT(1,1,N)=GT(1,1,N)+CF*SA*(R1**2-R2**2)/2.
   GT(1,2,N)=GT(1,2,N) +CF*SA*(R1**3-R2**3)/3.0
   GT(1,3,N)=GT(1,3,N)+CF*SA*(ALF12*(R1**3-R2**3)/3.0+0.5
  1*BET12*(R1**2-R2**2))
   GT(2,1,N)=GT(1,2,N)
   GT(2,2,N)=GT(2,2,N)+CF*SA*0.25*(R1**4-R2**4)
   GT(2,3,N)=GT(2,3,N)+CF*SA*(0.25*ALF12*(R1**4-R2**4)
  1 +BET12*(R1**3-R2**3)/3.0)
   GT(3,1,N)=GT(1,3,N)
   GT(3,2,N)=GT(2,3,N)
   GT(3,3,N)=GT(3,3,N)+CF*SA*(0.25*ALF12**2*(R1**4-R2**4)+2.0*ALF12
  1 *BET12*(R1**3-R2**3)/3.0+0.5*BET12**2*(R1**2-R2**2))
```

```
      GQ(1)=CF*SA*0.5*(R1**2-R2**2)
      GQ(2)=CF*SA*(R1**3-R2**3)/3.0
      GQ(3)=CF*SA*(ALF12*(R1**3-R2**3)/3.0+0.5*BET12*(R1**2-R2**2))
   51 CONTINUE
      DO 48 L1=1,3
      QR(I)=QR(I)+A(L1,2)*GQ(L1)
      IF(I.EQ.IM)GO TO 48
      QR(I+1)=QR(I+1)+A(L1,1)*GQ(L1)
   48 CONTINUE
      DO 12 L1=1,3
      DO 12 L2=1,3
      DO 12 L3=1,3
      DO 12 L4=1,3
      DO 12 L5=1,2
   12 CT(L1,L2,L5)=CT(L1,L2,L5)+A(L3,L1)*GT(L3,L4,L5)*A(L4,L2)
      DQ(K)=DQ(K)+.5*GT(2,2,2)*CP(MT)/CK(2,MT)
      IF(K.EQ.4)GO TO 31
      DQ(K+1)=DQ(K+1)+.5*GT(2,2,2)*CP(MT)/CK(2,MT)
      GO TO 32
   31 DQ(1)=DQ(1)+.5*GT(2,2,2)*CP(MT)/CK(2,MT)
32    CONTINUE
      DO 14 L1=1,2
      CQ(K,K,L1)=CQ(K,K,L1)+CT(1,1,L1)
      CQ(K,5,L1)=CQ(K,5,L1)+CT(1,3,L1)
      IF(K.EQ.4)GO TO 30
      CQ(K,K+1,L1)=CT(1,2,L1)
      CQ(K+1,K+1,L1)=CQ(K+1,K+1,L1)+CT(2,2,L1)
      CQ(K+1,5,L1)=CQ(K+1,5,L1)+CT(2,3,L1)
      GO TO 13
30    CQ(1,1,L1)=CQ(1,1,L1)+CT(2,2,L1)
      CQ(1,4,L1)=CT(1,2,L1)
      CQ(1,5,L1)=CQ(1,5,L1)+CT(2,3,L1)
   13 CQ(5,5,L1)=CQ(5,5,L1)+CT(3,3,L1)
   14 CONTINUE
      IF(J.NE.1)GO TO 15
      IF(K.NE.1)GO TO 15
      IF(I.EQ.IM)GO TO 15
      SAR=SQRT(1.+ALF12**2)/(R1-R2)
      QB(I,I)= SAR*(R2**2+R1*R2-2.0*R1**2)/6.0
      QB(I+1,I)= SAR*(2.0*R2**2-R1*R2-R1**2)/6.0
   15 CONTINUE
      DO 19 L1=1,2
      DO 18 L2=2,5
      L4=L2-1
       DO 18 L3=1,L4
   18 CQ(L2,L3,L1)=CQ(L3,L2,L1)
      DO 19 L2=1,4
      DO 19 L3=1,4
   19 CQ(L2,L3,L1)=CQ(L2,L3,L1)-CQ(L2,5,L1)*CQ(L3,5,L1)/CQ(5,5,L1)
      DO 40  L3=1,2
      F1(J,J,L3)=F1(J,J,L3)+CQ(1,1,L3)
      F1(J,J+1,L3)=F1(J,J+1,L3)+CQ(1,4,L3)
      F1(J+1,J+1,L3)=F1(J+1,J+1,L3)+CQ(4,4,L3)
      F2(J,J,L3)=F2(J,J,L3)+CQ(2,2,L3)
      F2(J,J+1,L3)=F2(J,J+1,L3)+CQ(2,3,L3)
      F2(J+1,J+1,L3)=F2(J+1,J+1,L3)+CQ(3,3,L3)
      G(J,J,L3)=G(J,J,L3)+CQ(1,2,L3)
```

```
      G(J,J+1,L3)=G(J,J+1,L3)+CQ(1,3,L3)
      G(J+1,J,L3)=G(J+1,J,L3)+CQ(4,2,L3)
      G(J+1,J+1,L3)=G(J+1,J+1,L3)+CQ(4,3,L3)
   40 CONTINUE
      D1(J)=D1(J)+DQ(1)
      D1(J+1)=D1(J+1)+DQ(4)
      D2(J)=D2(J)+DQ(2)
      D2(J+1)=D2(J+1)+DQ(3)
   21 CONTINUE
      DO 22 L3=1,2
      DO 22 L1=1,JM
      DO 22 L2=L1,JM
      IR=JM*(I-1)
      LR=IR+L1
      LC=IR+L2
      C(LR,LC,L3)=F1(L1,L2,L3)
   22 F1(L1,L2,L3)=F2(L1,L2,L3)
      DO 26 L1=1,JM
      IR=JM*(I-1)
      LR=IR+L1
      D(LR)=D1(L1)
   26 D1(L1)=D2(L1)
      IF(I.EQ.IM)GO TO 24
      DO 23 L3=1,2
      DO 23 L1=1,JM
      DO 23 L2=1,JM
      IR=JM*(I-1)
      LR=IR+L1
      LC=IR+JM+L2
   23 C(LR,LC,L3)=G(L1,L2,L3)
   24 CONTINUE
      DO 27 L3=1,2
      DO 27 L1=2,M
      L1M=L1-1
      DO 27 L2=1,L1M
   27 C(L1,L2,L3)=C(L2,L1,L3)
      RETURN
      END
```

```
      SUBROUTINE TRIA(NEQ,M,A)
      DIMENSION A(1)
      NE=NEQ-1
      MN=M-1
      MM=MN*NEQ
      MK=NEQ-MN
      DO 300 N=1,NE
      NT=N-MK
      IF(NT.GT.0) MM=MM-NEQ
      IF(A(N).EQ.0.0) GO TO 300
      L=N
      IL=N+NEQ
      IH=N+MM
      DO 200 I=IL,IH,NEQ
      L=L+1
      J=L
   90 C=A(I)/A(N)
      DO 100 K=I,IH,NEQ
      A(J)=A(J)-C*A(K)
  100 J=J+NEQ
      A(I)=C
  200 CONTINUE
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INTGRL(R,Z,X)
      REAL ICON,ICONP,IZ,IZP,IZ2,IZ2P
      DIMENSION R(3),Z(3),X(10),XI(10),AI(10)
      RI=R(1)
      RJ=R(2)
      RK=R(3)
      DATA(XI(I),AI(I),I=1,10)/-.97390653,.066671344,-.86506337,.1494513
     15,-.67940957,.21908636,-.43339539,.26926672,-.14887434,.29552422,
     2.14887434,.29552422,.43339539,.26926672,.67940957,.21908636,
     3.86506337,.14945135,.97390653,.066671344/
      DO 2001 N1=1,10
 2001 X(N1)=0.
C****  CALCULATION OF INTEGRALS BY GAUSSIAN QUADRATURE
   70 RMIN=AMIN1(RI,RJ,RK)
      RMAX=AMAX1(RI,RJ,RK)
      DO 7 N1=1,3
    7 IF(ABS(R(N1)-RMIN).LE.0.00001)I1=N1
      DO 8 N1=1,3
    8 IF(ABS(R(N1)-RMAX).LE.0.00001)I3=N1
      DO 9 N1=1,3
    9 IF(N1.NE.I1.AND.N1.NE.I3) I2=N1
       R1=R(I1)
      R2=R(I2)
      R3=R(I3)
      Z1=Z(I1)
      Z2=Z(I2)
      Z3=Z(I3)
      FAC=1.0
      DR12=ABS(R1-R2)
      DR13=ABS(R1-R3)

      S12=(Z2-Z1)/(R2-R1)
      S13=(Z3-Z1)/(R3-R1)
      S23=(Z3-Z2)/(R3-R2)
      DR=R2-R1
      DRP=R3-R2
      DO 12 N1=1,10
      RR=R1+DR*(XI(N1)+1.)/2.
      RRP=R2+DRP*(XI(N1)+1.)/2.
      ZZ1=S13*(RR-R1)+Z1
      ZZ1P=S13*(RRP-R1)+Z1
      ZZ2=S12*(RR-R1)+Z1
      ZZ3=S23*(RRP-R2)+Z2
       ICON=ABS(ZZ2-ZZ1)
       ICONP=ABS(ZZ3-ZZ1P)
      IZ=(ZZ1**2-ZZ2**2)/2.
      IF(ZZ1.LT.ZZ2) IZ= -IZ
      IZP=(ZZ1P**2-ZZ3**2)/2.
      IF(ZZ1P.LT.ZZ3) IZP= -IZP
      IZ2=ABS(ZZ2**3-ZZ1**3)/3.
      IZ2P=ABS(ZZ3**3-ZZ1P**3)/3.
      DO 10 N2=1,5
      X(N2)=X(N2)+AI(N1)*ICONP*RRP**(N2-2)*DRP
   10 IF(ABS(RR).GT.0.0000001)
     1X(N2)=X(N2)+AI(N1)*(ICON*RR**(N2-2)*DR)
      DO 11 N2=6,9
```

```
      X(N2)=X(N2)+AI(N1)*IZP*RRP**(N2-7)*DRP
 11   IF(ABS(RR).GT.0.0000001)
     1X(N2)=X(N2)+AI(N1)*(IZ*RR**(N2-7)*DR)
 12   X(10)=X(10)+AI(N1)*(IZ2/RR*DR+IZ2P/RRP*DRP)
      DO 13 N1=1,10
 13   X(N1)=X(N1)/2.
      X(1)=FAC*X(1)
      X(6)=FAC*X(6)
      X(10)=FAC*X(10)
      RETURN
      END
```

```
      SUBROUTINE DATA(JM)
      DIMENSION E(3,3),V(3,3),PD(15,5,15),PS(15,5,14),PL(15,5,14),
     1  A(75,75)
      DIMENSION CN(3,3,3),G(3,3),ALF(3,3),CK(3,3),JL(2),CP(3)
      COMMON/DAT/CN,G,ALF,CK,JL,CP
      COMMON/STIF1/PD,PS,PL,A
      DO 1 M=1,3
      READ(5,100)E(1,M),E(2,M),E(3,M),G(1,M),G(2,M),G(3,M),
     1  V(1,M),V(2,M),V(3,M)
    1 READ(5,110)ALF(1,M),ALF(2,M),ALF(3,M),CK(1,M),CK(2,M),CK(3,M),CP(M
     1)
  100 FORMAT(9F8.5)
  110 FORMAT(7E8.5)
      DO 2 M=1,3
    2 WRITE(6,101)M,E(1,M),E(2,M),E(3,M),G(1,M),G(2,M),G(3,M),
     1  V(1,M),V(2,M),V(3,M),ALF(1,M),ALF(2,M),ALF(3,M),
     2CK(1,M),CK(2,M),CK(3,M),CP(M)
  101 FORMAT(///13H MATERIAL NO.,I2/
     1 7H ER   = ,E12.5,5X,6HET   = ,E12.5,5X,6HEZ   = ,E12.5/
     2 7H GRT  = ,E12.5,5X,6HGRZ  = ,E12.5,5X,6HGTZ  = ,E12.5/
     3 7H VTR  = ,E12.5,5X,6HVZR  = ,E12.5,5X,6HVZT  = ,E12.5/
     4 7H AR   = ,E12.5,5X,6HAT   = ,E12.5,5X,6HAZ   = ,E12.5/
     5 7H KR   = ,E12.5,5X,6HKT   = ,E12.5,5X,6HKZ   = ,E12.5/
     67H CP   = ,E12.5///)
      DO 3 M=1,3
      A(1,1)=1./E(1,M)
      A(1,2)=-V(1,M)/E(2,M)
      A(1,3)=-V(2,M)/E(3,M)
      A(2,2)=1./E(2,M)
      A(2,3)=-V(3,M)/E(3,M)
      A(3,3)=1./E(3,M)
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      CALL  INVERT(A,3,75)
      DO 4  I=1,3
      DO 4  J=1,3
    4 CN(I,J,M)=A(I,J)
    3 CONTINUE
      READ(5,102)(JL(I),I=1,2)
  102 FORMAT(2I5)
      WRITE(6,130)JL(1),JL(1),JL(2),JL(2),JM
  130 FORMAT(//21H LAYER NO.1 J = 1 TO ,I2/
     1    15H LAYER NO.2 J =,I2,4H TO ,I2/
     2    15H LAYER NO.3 J =,I2,4H TO ,I2//)
      RETURN
      END
```

```
      SUBROUTINE INVERT(D,ACT,DIM)
C     INVERSION OF SYMMETRIC MATRIX
      INTEGER ACT,DIM
      DIMENSION D(DIM,DIM),LOC(76)
      DOUBLE PRECISION DP
      DP=1.D0
      DO 1 N=1,ACT
    1 LOC(N)=N
      DO 6 N1=1,ACT
      M=0
      PIVOT=0.
      DO 2 N2=N1,ACT
      NN=LOC(N2)
      IF (ABS(D(NN,NN)).LE.ABS(PIVOT)) GO TO 2
      M=N2
      PIVOT=D(NN,NN)
    2 CONTINUE
      IF (M.EQ.0) GO TO 8
      N=LOC(M)
      LOC(M)=LOC(N1)
      LOC(N1)=N
      D(N,N)=-1.
      DO 3 J=1,ACT
    3 D(N,J)=D(N,J)/PIVOT
      DO 5 I1=1,ACT
      I=LOC(I1)
      IF (N.EQ.I.OR.D(I,N).EQ.0.) GO TO 5
      DO 4 J1=I1,ACT
      J=LOC(J1)
      IF (N.EQ.J) GO TO 4
      D(I,J)=D(I,J)-D(I,N)*D(N,J)*DP
      D(J,I)=D(I,J)
    4 CONTINUE
    5 CONTINUE
      DO 6 I=1,ACT
    6 D(I,N)=D(N,I)
      DO 7 I=1,ACT
      DO 7 J=1,ACT
    7 D(I,J)=-D(I,J)
      RETURN
    8 WRITE(6,9)
    9 FORMAT (42H0MATRIX IS SINGULAR - EXECUTION TERMINATED   )
      STOP
      END
```

```
      SUBROUTINE BACKS(NN,MM,A,B)
C
      DIMENSION A(1),B(1)
C
      MMM=MM-1
      N=0
  270 N=N+1
      C=B(N)
      IF(A(N).NE.0.0) B(N)=B(N)/A(N)
      IF(N.EQ.NN) GO TO 300
      IL=N+1
      IH=MINO(NN,N+MMM)
      M=N
      DO 285 I=IL,IH
      M=M+NN
  285 B(I)=B(I)-A(M)*C
      GO TO 270
C
  300 IL=N
      N=N-1
      IF(N.EQ.0) RETURN
      IH=MINO(NN,N+MMM)
      M=N
      DO 400 I=IL,IH
      M=M+NN
  400 B(N)=B(N)-A(M)*B(I)
      GO TO 300
C
      END
```

```
      SUBROUTINE STIFF (IM,JM,N,CF)
      DIMENSION CN(3,3,3),G(3,3),ALF(3,3),CK(3,3),JL(2),SP(3)
      COMMON/DAT/CN,G,ALF,CK,JL,SP
      COMMON R,Z
      COMMON/STIF1/PD,PS,PL,D
      COMMON/STIF2/KL,KD,KS
      REAL KD,KT,KQ,KS
      DIMENSION R(15,5),Z(15,5),KD(15,15,15),KS(15,15,15),PD(15,5,15),
     1PS(15,5,14),PL(15,5,14),D(75,75)
      DIMENSION A(3,3),NN(10),RT(9,3),KT(9,9),KQ(15,15),PT(9,3),
     2PQ(15,5),GT(3,3,2),CQ(5,5,2),CP(5,5),DQ(4),GQ(3),CT(3,3,2)
      DIMENSION RZ(3),ZR(3),X(10)
      DIMENSION KL(15,15,15)
      FN=N
      IM1=IM-1
      JM1=JM-1
      NN(1)=1
      NN(2)=3
      NN(3)=4
      NN(4)=6
      NN(5)=7
      NN(6)=9
      NN(7)=10
      NN(8)=12
      NN(9)=13
      NN(10)=15
C
C     INITIALIZE
C
      DO 101 L1=1,15
      DO 101 L2=1,15
      DO 101 L3=1,15
      KS(L1,L2,L3)=0.0
  101 KD(L1,L2,L3)=0.0
      DO 102 L1=1,15
      DO 102 L2=1,5
      DO 102 L3=1,15
  102 PD(L1,L2,L3)=0.0
      DO 222 L1=1,15
      DO 222 L2=1,5
      DO 222 L3=1,14
      PL(L1,L2,L3)=0.0
  222 PS(L1,L2,L3)=0.0
C
C     OUTER LOOP ON I BEGINS HERE
C
      DO 402 I=1,IM1
C
C     INNER LOOP ON J BEGINS HERE
C
      DO 21 J=1,JM1
      IF(J.LT.JL(1))MT=1
      IF(J.GE.JL(1).AND.J.LT.JL(2))MT=2
      IF(J.GE.JL(2))MT=3
C     INITIALIZE FOR I,J QUAD
      DO 104 L1=1,15
```

```
      DO 104 L2=1,15
 104  KQ(L1,L2)=0.0
      DO 105 L1=1,15
      DO 105 L2=1,5
  105 PQ(L1,L2)=0.0
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      R3=R(I+1,J+1)
      Z3=Z(I+1,J+1)
      R4=R(I,J+1)
      Z4=Z(I,J+1)
      R21=R2-R1
      R32=R3-R2
      R41=R4-R1
      R34=R3-R4
      Z21=Z2-Z1
      Z32=Z3-Z2
      Z41=Z4-Z1
      Z34=Z3-Z4
      AREA=.5*(R41*Z41-R21*Z21-R32*Z32+R34*Z34)-R32*Z21+R34*Z41
      AR=.5*(R41*Z41)*(R1+2.*R41/3.0)-.5*R21*Z21*(R1+2.*R21/3.)-R32*Z21*
     3(R2+.5*R32)-.5*R32*Z32*(R2+2.*R32/3.)+.5*R34*Z34*(R4+2.*R34/3.)+R3
     44*Z41*(R4+.5*R34)
      AZ=.5*R41*Z41*(Z1+Z41/3.)-.5*R21*Z21*(Z1+Z21/3.)-R32*Z21*(Z1+.5*Z2
     51)-.5*R32*Z32*(Z2+Z32/3.)+.5*R34*Z34*(Z4+Z34/3.)+R34*Z41*(Z1+.5*Z4
     61)
      RC=AR/AREA
      ZC=AZ/AREA
      DO 16 L1=1,5
      DO 16 L2=1,5
      DO 16 L3=1,2
   16 CQ(L1,L2,L3)=0.0
      DO 17 L1=1,4
   17 DQ(L1)=0.0
      DO 15 K=1,4
      DO 601 L1=1,9
      DO 601 L2=1,9
 601  KT(L1,L2)=0.0
      DO 602 L1=1,9
      DO 602 L2=1,3
 602  PT(L1,L2)=0.0
      IF(K.NE.1)GO TO 1
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      GO TO 4
    1 IF(K.NE.2)GO TO 2
      R1=R(I+1,J)
      Z1=Z(I+1,J)
      R2=R(I+1,J+1)
      Z2=Z(I+1,J+1)
      GO TO 4
    2 IF(K.NE.3)GO TO 3
      R1=R(I+1,J+1)
```

```
      Z1=Z(I+1,J+1)
      R2=R(I,J+1)
      Z2=Z(I,J+1)
      GO TO 4
    3 IF(K.NE.4)GO TO 4
      R1=R(I,J+1)
      Z1=Z(I,J+1)
      R2=R(I,J)
      Z2=Z(I,J)
    4 R3=RC
      Z3=ZC
      AT=0.5*(R2*Z3-R3*Z2+R3*Z1-R1*Z3+R1*Z2-R2*Z1)
      A(1,1)=.5*(R2*Z3-R3*Z2)/AT
      A(2,1)=.5*(Z2-Z3)/AT
      A(3,1)=.5*(R3-R2)/AT
      A(1,2)=.5*(R3*Z1-R1*Z3)/AT
      A(2,2)=.5*(Z3-Z1)/AT
      A(3,2)=.5*(R1-R3)/AT
      A(1,3)=.5*(R1*Z2-R2*Z1)/AT
      A(2,3)=.5*(Z1-Z2)/AT
      A(3,3)=.5*(R2-R1)/AT
      DO 5 L1=1,3
      GQ(L1)=0.0
      DO 5 L2=1,3
      DO 5 L3=1,2
      GT(L1,L2,L3)=0.0
    5 CT(L1,L2,L3)=0.0
      IF(ABS(R1-R2).LT.0.000001)GO TO 6
      ALF12=(Z1-Z2)/(R1-R2)
      BET12=(Z2*R1-Z1*R2)/(R1-R2)
      GO TO 7
    6 ALF12=0.0
      BET12=0.0
    7 IF(ABS(R2-R3).LT.0.000001)GO TO 8
      ALF23=(Z2-Z3)/(R2-R3)
      BET23=(Z3*R2-Z2*R3)/(R2-R3)
      GO TO 9
    8 ALF23=0.
      BET23=0.
    9 IF(ABS(R3-R1).LT.0.000001)GO TO 10
      ALF31=(Z3-Z1)/(R3-R1)
      BET31=(Z1*R3-Z3*R1)/(R3-R1)
      GO TO 11
   10 ALF31=0.0
      BET31=0.0
   11 CONTINUE
      RZ(1)=R1
      RZ(2)=R2
      RZ(3)=R3
      ZR(1)=Z1
      ZR(2)=Z2
      ZR(3)=Z3
      CALL INTGRL(RZ,ZR,X)
      GT(2,2,1)=X(3)
      GT(3,3,1)=X(3)
      GT(1,1,2)=X(1)
      GT(1,2,2)=X(2)
```

```
      GT(1,3,2)=X(6)
      GT(2,1,2)=X(2)
      GT(2,2,2)=X(3)
      GT(2,3,2)=X(7)
      GT(3,1,2)=X(6)
      GT(3,2,2)=X(7)
      GT(3,3,2)=X(10)
      R314=R3**4-R1**4
      R313=R3**3-R1**3
      R234=R2**4-R3**4
      R233=R2**3-R3**3
      R312=R3**2-R1**2
      R232=R2**2-R3**2
      GD1=0.25*(ALF31-ALF12)*R314+(BET31-BET12)*R313/3.0+0.25*(ALF23-
     4ALF12)*R234+(BET23-BET12)*R233/3.0
      GD2=0.125*(ALF31**2-ALF12**2)*R314+(ALF31*BET31-ALF12*BET12)*     R
     5313/3.0+.25*(BET31**2-BET12**2)*R312+0.125*(ALF23**2-ALF12**2)*
     6R234+(ALF23*BET23-ALF12*BET12)*R233/3.0+.25*(BET23**2-BET12**2)*
     2R232
      GD3=(ALF31**3-ALF12**3)*R314/12.0+(ALF31**2*BET31-ALF12**2*BET12
     6)*R313/3.0+0.5*(ALF31*BET31**2-ALF12*BET12**2)*R312+(BET31**3-  BE
     7T12**3)*(R3-R1)/3.0+(ALF23**3-ALF12**3)*R234/12.0+(ALF23**2*BET23-
     8ALF12**2*BET12)*R233/3.0+0.5*(ALF23*BET23**2-ALF12*BET12**2)*R232+
     9(BET23** 3-BET12**3)*(R2-R3)/3.0
C     FORM KT AND PT FOR TRIANGLE
C     INSERT A
      Y1=1.
      Y2=1.
      IF(N.EQ.0)Y1=2.
      IF(N.EQ.0)Y2=0.
      DO 209 I1=1,3
      DO 209 J1=1,3
      AI=A(1,I1)
      BI=A(2,I1)
      DI=A(3,I1)
      AJ=A(1,J1)
      BJ=A(2,J1)
      DJ=A(3,J1)
      GG1=GT(2,2,1)
      GG2=AJ*GT(1,2,2)+BJ*GG1+DJ*GT(2,3,2)
      GG3=AI*GT(1,2,2)+BI*GG1+DI*GT(2,3,2)
      GG4=AI*AJ*GT(1,1,2)+(AI*BJ+AJ*BI)*GT(1,2,2)+(AI*DJ+AJ*DI)*GT(
     11,3,2)+BI*BJ*GG1+(BI*DJ+BJ*DI)*GT(2,3,2)+DI*DJ*GT(3,3,2)
      GG5=AI*GG1+BI*GD1+DI*GD2
      GG6=AI*AJ*GT(1,2,2)+(AI*BJ+AJ*BI)*GG1+(AI*DJ+AJ*DI)*GT(2,3,2)+
     1BI*BJ*GD1+(BI*DJ+BJ*DI)*GD2+DI*DJ*GD3
      GG7=AJ*GG1+BJ*GD1+DJ*GD2
      DO 208 L1=1,3
      DO 208 L2=1,3
      LR=(I1-1)*3+L1
      LC=(J1-1)*3+L2
      IF(L1.NE.1)GO TO 202
      IF(L2.NE.1)GO TO 200
      PT(LR,J1)=Y1*((ALF(1,MT)*CN(1,1,MT)+ALF(2,MT)*CN(1,2,MT)
     1+ALF(3,MT)*CN(1,3,MT))*BI*GG7+(ALF(1,MT)*CN(2,1,MT)+ALF(2,MT)*
     2CN(2,2,MT)+ALF(3,MT)*CN(2,3,MT))*GG6)
      KT(LR,LC)=Y1*(CN(1,1,MT)*BI*BJ*X(3)+CN(1,2,MT)*BI*GG2+CN(2,1,MT)
```

```
     1*BJ*GG3+CN(2,2,MT)*GG4+G(2,MT)*DI*DJ*X(3))+Y2*FN*FN*G(1,MT)*GG4
      GO TO 208
  200 IF(L2.NE.2)GO TO 201
      KT(LR,LC)=Y1*FN*(CN(1,2,MT)*BI*GG2+CN(2,2,MT)*GG4)+Y2*FN*G(1,MT)*
     1(GG4-BJ*GG3)
      GO TO 208
  201 KT(LR,LC)=Y1*(CN(1,3,MT)*BI*DJ*X(3)+CN(2,3,MT)*DJ*GG3
     1+G(2,MT)*BJ*DI*X(3))
      GO TO 208
  202 IF(L1.NE.2)GO TO 205
      IF(L2.NE.1)GO TO 203
      PT(LR,J1)=Y1*FN*(ALF(1,MT)*CN(2,1,MT)+ALF(2,MT)*CN(2,2,MT)+ALF(3
     1,MT)*CN(2,3,MT))*GG6
      KT(LR,LC)=Y1*FN*(CN(2,1,MT)*BJ*GG3+CN(2,2,MT)*GG4)+Y2*FN*
     1G(1,MT)*(GG4-BI*GG2)
      GO TO 208
  203 IF(L2.NE.2)GO TO 204
      KT(LR,LC)=Y1*FN*FN*CN(2,2,MT)*GG4+Y2*(G(1,MT)*BI*BJ*X(3)-G(1,MT)
     1*(BI*GG2+BJ*GG3-GG4)+G(3,MT)*DI*DJ*X(3))
      GO TO 208
  204 KT(LR,LC)=Y1*FN*CN(2,3,MT)*DJ*GG3-Y2*FN*G(3,MT)*DI*GG2
      GO TO 208
  205 IF(L2.NE.1)GO TO 206
      PT(LR,J1)=Y1*(ALF(1,MT)*CN(3,1,MT)+ALF(2,MT)*CN(3,2,MT)+ALF(3,MT)
     1*CN(3,3,MT))*DI*GG7
      KT(LR,LC)=Y1*(CN(3,1,MT)*BJ*DI*X(3)+CN(3,2,MT)*DI*GG2+G(2,MT)
     1*BI*DJ*X(3))
      GO TO 208
  206 IF(L2.NE.2)GO TO 207
      KT(LR,LC)=Y1*FN*CN(3,2,MT)*DI*GG2-Y2*FN*G(3,MT)*DJ*GG3
      GO TO 208
  207 KT(LR,LC)=Y1*(CN(3,3,MT)*DI*DJ*X(3)+G(2,MT)*BI*BJ*X(3))
     1+Y2*FN*FN*G(3,MT)*GG4
  208 CONTINUE
  209 CONTINUE
C
C     FORM CT
C
      DO 50 L1=1,3
      DO 50 L2=1,3
   50 GT(L1,L2,2)=GT(L1,L2,2)*CK(2,MT)
      GT(2,2,1)=GT(2,2,1)*CK(1,MT)
      GT(3,3,1)=GT(3,3,1)*CK(3,MT)
      IF(J.NE.JM1)GO TO 51
      IF(K.NE.3)GO TO 51
      SA=SQRT(1.+ALF12**2)
      GT(1,1,1)=GT(1,1,1)+CF*SA*(R1**2-R2**2)/2.
      GT(1,2,1)=GT(1,2,1)+CF*SA*(R1**3-R2**3)/3.
      GT(1,3,1)=GT(1,3,1)+CF*SA*(ALF12*(R1**3-R2**3)/3.+.5*BET12*(R1**2
     3-R2**2))
      GT(2,1,1)=GT(1,2,1)
      GT(2,2,1)=GT(2,2,1)+CF*SA* .25*(R1**4-R2**4)
      GT(2,3,1)=GT(2,3,1)+CF*SA*(0.25*ALF12*(R1**4-R2**4)+BET12*(R1**3
     1 -R2**3)/3.0)
      GT(3,1,1)=GT(1,3,1)
      GT(3,2,1)=GT(2,3,1)
      GT(3,3,1)=GT(3,3,1)+CF*SA*(0.25*ALF12**2*(R1**4-R2**4)+2.0*ALF12
```

```
    1 *BET12*(R1**3-R2**3)/3.0+0.5*BET12**2*(R1**2-R2**2))
   51 CONTINUE
      DO 12 L1=1,3
      DO 12 L2=1,3
      DO 12 L3=1,3
      DO 12 L4=1,3
      DO 12 L5=1,2
   12 CT(L1,L2,L5)=CT(L1,L2,L5)+A(L3,L1)*GT(L3,L4,L5)*A(L4,L2)
C     NOW FOR THE QUADRILATERAL
C
      DO 300 K1=1,3
      DO 300 K2=1,3
      KR=3*(K-1)+K1
      KC=3*(K-1)+K2
      KR5=12+K1
      KC5=12+K2
      KRR=3*K+K1
      KCC=3*K+K2
      KQ(KR,KC)=KQ(KR,KC)+KT(K1,K2)
      KQ(KR,KC5)=KQ(KR,KC5)+KT(K1,K2+6)
      KQ(KR5,KC)=KQ(KR5,KC)+KT(K1+6,K2)
      IF(K.EQ.4) GO TO 301
      KQ(KR,KCC)=KQ(KR,KCC)+KT(K1,K2+3)
      KQ(KRR,KC)=KQ(KRR,KC)+KT(K1+3,K2)
      KQ(KRR,KCC)=KQ(KRR,KCC)+KT(K1+3,K2+3)
      KQ(KRR,KC5)=KQ(KRR,KC5)+KT(K1+3,K2+6)
      KQ(KR5,KCC)=KQ(KR5,KCC)+KT(K1+6,K2+3)
      GO TO 302
  301 KQ(K1,K2)=KQ(K1,K2)+KT(K1+3,K2+3)
      KQ(K1,K2+9)=KQ(K1,K2+9)+KT(K1+3,K2)
      KQ(K1+9,K2)=KQ(K1+9,K2)+KT(K1,K2+3)
      KQ(KRR,K2)=KQ(KRR,K2)+KT(K1+6,K2+3)
      KQ(K1,KCC)=KQ(K1,KCC)+KT(K1+3,K2+6)
  302 KQ(KR5,KC5)=KQ(KR5,KC5)+KT(K1+6,K2+6)
  300 CONTINUE
      DO 303 K1=1,3
      KR=3*(K-1)+K1
      PQ(KR,K)=PQ(KR,K)+PT(K1,1)
      PQ(KR,5)=PQ(KR,5)+PT(K1,3)
      PQ(K1+12,K)=PQ(K1+12,K)+PT(K1+6,1)
      IF(K.EQ.4) GO TO 304
      PQ(KR,K+1)=PQ(KR,K+1)+PT(K1,2)
      PQ(KR+3,K)=PQ(KR+3,K)+PT(K1+3,1)
      PQ(KR+3,K+1)=PQ(KR+3,K+1)+PT(K1+3,2)
      PQ(KR+3,5)=PQ(KR+3,5)+PT(K1+3,3)
      PQ(K1+12,K+1)=PQ(K1+12,K+1)+PT(K1+6,2)
      GO TO 305
  304 PQ(K1,4)=PQ(K1,4)+PT(K1+3,1)
      PQ(K1,5)=PQ(K1,5)+PT(K1+3,3)
      PQ(K1+9,1)=PQ(K1+9,1)+PT(K1,2)
      PQ(K1,1)=PQ(K1,1)+PT(K1+3,2)
      PQ(K1+12,1)=PQ(K1+12,1)+PT(K1+6,2)
  305 PQ(K1+12,5)=PQ(K1+12,5)+PT(K1+6,3)
  303 CONTINUE
      DO 14 L1=1,2
      CQ(K,K,L1)=CQ(K,K,L1)+CT(1,1,L1)
      CQ(K,5,L1)=CQ(K,5,L1)+CT(1,3,L1)
```

```
      IF(K.EQ.4)GO TO 30
      CQ(K,K+1,L1)=CT(1,2,L1)
      CQ(K+1,K+1,L1)=CQ(K+1,K+1,L1)+CT(2,2,L1)
      CQ(K+1,5,L1)=CQ(K+1,5,L1)+CT(2,3,L1)
      GO TO 13
   30 CQ(1,1,L1)=CQ(1,1,L1)+CT(2,2,L1)
      CQ(1,4,L1)=CT(1,2,L1)
      CQ(1,5,L1)=CQ(1,5,L1)+CT(2,3,L1)
   13 CQ(5,5,L1)=CQ(5,5,L1)+CT(3,3,L1)
   14 CONTINUE
      DO 18 L1=1,2
      DO 19 L2=2,5
      L4=L2-1
      DO 19 L3=1,L4
   19 CQ(L2,L3,L1)=CQ(L3,L2,L1)
   18 CONTINUE
   15 CONTINUE
      DO 500 L1=1,5
      DO 500 L2=1,5
  500 CP(L1,L2)=CQ(L1,L2,1)+FN*FN*CQ(L1,L2,2)
C
C     NEED TO ELIMINATE MIDDLE NODE
C
C     INSERT A
      N1=3
      IF(N.NE.0)GO TO 801
      N1=2
      DO 800 M1=1,10
      M3=NN(M1)
      DO 802 M5=1,5
  802 PQ(M1,M5)=PQ(M3,M5)
      DO 800 M2=1,10
      M4=NN(M2)
  800 KQ(M1,M2)=KQ(M3,M4)
  801 CONTINUE
      N2=2*N1
      N3=3*N1
      N4=4*N1
      DO 306 K1=1,N1
      DO 306 K2=1,N1
  306 D(K1,K2)=KQ(K1+N4,K2+N4)
      CALL INVERT(D,N1,75)
      DO 307 K1=1,N4
      DO 307 K2=1,N4
      DO 307 K3=1,N1
      DO 307 K4=1,N1
      L3=K3+N4
      L4=K4+N4
  307 KQ(K1,K2)=KQ(K1,K2)-KQ(K1,L3)*D(K3,K4)*KQ(L4,K2)
      DO 309 L1=1,N4
      DO 309 L2=1,4
  309 PQ(L1,L2)=PQ(L1,L2)-PQ(L1,5)*CP(5,L2)/CP(5,5)
      DO 308 L1=1,N1
      DO 308 L2=1,4
  308 PQ(N4+L1,L2)=PQ(N4+L1,L2)-PQ(N4+L1,5)*CP(5,L2)/CP(5,5)
      DO 310 L1=1,N4
      DO 310 L2=1,4
```

```
      DO 310 K1=1,N1
      DO 310 K2=1,N1
  310 PQ(L1,L2)=PQ(L1,L2)-KQ(L1,K1+N4)*D(K1,K2)*PQ(N4+K2,L2)
C
C     ASSEMBLE THE ROW MATRICES KD,KS,PD,PS
C
      DO 400 K1=1,N1
      DO 400 K2=1,N1
      KR=N1*(J-1)+K1
      KC=N1*(J-1)+K2
      KD(KR,KC,I)=KD(KR,KC,I)+KQ(K1,K2)
      KD(KR,KC+N1,I)=KD(KR,KC+N1,I)+KQ(K1,K2+N3)
      KD(KR+N1,KC,I)=KD(KR+N1,KC,I)+KQ(K1+N3,K2)
      KD(KR+N1,KC+N1,I)=KD(KR+N1,KC+N1,I)+KQ(K1+N3,K2+N3)
      KD(KR,KC,I+1)=KD(KR,KC,I+1)+KQ(K1+N1,K2+N1)
      KD(KR,KC+N1,I+1)=KD(KR,KC+N1,I+1)+KQ(K1+N1,K2+N2)
      KD(KR+N1,KC,I+1)=KD(KR+N1,KC,I+1)+KQ(K1+N2,K2+N1)
      KD(KR+N1,KC+N1,I+1)=KD(KR+N1,KC+N1,I+1)+KQ(K1+N2,K2+N2)
      KS(KR,KC,I)=KS(KR,KC,I)+KQ(K1,K2+N1)
      KS(KR,KC+N1,I)=KS(KR,KC+N1,I)+KQ(K1,K2+N2)
      KS(KR+N1,KC,I)=KS(KR+N1,KC,I)+KQ(K1+N3,K2+N1)
      KS(KR+N1,KC+N1,I)=KS(KR+N1,KC+N1,I)+KQ(K1+N3,K2+N2)
  400 CONTINUE
      DO 401 K1=1,N1
      KR=N1*(J-1)+K1
      PD(KR,J,I)=PD(KR,J,I)+PQ(K1,1)
      PD(KR,J+1,I)=PD(KR,J+1,I)+PQ(K1,4)
      PD(KR+N1,J,I)=PD(KR+N1,J,I)+PQ(K1+N3,1)
      PD(KR+N1,J+1,I)=PD(KR+N1,J+1,I)+PQ(K1+N3,4)
      PD(KR,J,I+1)=PD(KR,J,I+1)+PQ(K1+N1,2)
      PD(KR,J+1,I+1)=PD(KR,J+1,I+1)+PQ(K1+N1,3)
      PD(KR+N1,J,I+1)=PD(KR+N1,J,I+1)+PQ(K1+N2,2)
      PD(KR+N1,J+1,I+1)=PD(KR+N1,J+1,I+1)+PQ(K1+N2,3)
      PS(KR,J,I)=PS(KR,J,I)+PQ(K1,2)
      PS(KR,J+1,I)=PS(KR,J+1,I)+PQ(K1,3)
      PS(KR+N1,J,I)=PS(KR+N1,J,I)+PQ(K1+N3,2)
      PS(KR+N1,J+1,I)=PS(KR+N1,J+1,I)+PQ(K1+N3,3)
      PL(KR,J,I)=PL(KR,J,I)+PQ(K1+N1,1)
      PL(KR,J+1,I)=PL(KR,J+1,I)+PQ(K1+N1,4)
      PL(KR+N1,J,I)=PL(KR+N1,J,I)+PQ(K1+N2,1)
      PL(KR+N1,J+1,I)=PL(KR+N1,J+1,I)+PQ(K1+N2,4)
  401 CONTINUE
   21 CONTINUE
  402 CONTINUE
      RETURN
      END
```

APPENDIX B

DESCRIPTION OF CONTROL PROGRAM

Purpose

This program computes the patch heats necessary to minimize the weighted surface error of the mirror taken at designated sample points, using designated heater locations. In addition it computes the surface error both before and after application of the thermal input. The disturbance errors are computed internally.

Input Parameter Definition

Parameter	Definition
IM	Number of nodes in radial direction.
KM	Number of angular divisions.
DI	Inner diameter.
DO	Outer diameter.
FNO	Focal length/diameter.
NHP	Number of heater locations.
NSP	Number of sample points.
IHP	Individual heater location.
ISP	Individual sample point.
A(I,J)	Coefficients of the influence matrix computed in RESPONSE (read from file).

Input Data Card Listing

Card No.	Parameter	Data Field	Format
1	IM	1-5	I5
1	KM	6-10	I5
1	DI	11-20	F10.5
1	DO	21-30	F10.5
1	FNO	31-40	F10.5
2	NHP	1-5	I5
2	NSP	6-10	I5
3	IHP(I)	1-80	16I5
4	ISP(I)	1-80	16I5

Output of Program

1. Repeated heat patch points.
2. Repeated sample points.
3. Coefficients of the reduced influence matrix corresponding to the heat patches and sample points selected.
4. The surface error of the sample points before control.
5. The performance index before control.
6. The performance index after control.

Program Listing

A listing of the program appears on the following pages. The program calls subroutine INVERT which has been listed on earlier pages.

```
C
C
C
      DIMENSION IHP(50),ISP(180),A(180,168),B(50,50),C(50,180),D(50,50),
     1  WD(180),WW(180)
      DOUBLE PRECISION B
      BN=1.0
C     INITIALIZATION
      DO 12 I=1,180
      WD(I)=0.0
      WW(I)=0.0
      ISP(I)=0
      DO 11 J=1,50
   11 C(J,I)=0.0
      DO 12 J=1,168
   12 A(I,J)=0.0
      DO 20 I=1,50
      IHP(I)=0.0
      DO 20 J=1,50
   20 B(I,J)=0.0
      READ(5,50) IM,KM,DI,DO,FNO
   50 FORMAT(2I5,3F10.5)
      RI=DI/2.0
      RO=DO/2.0
COMMENT
C     INPUT SAMPLE POINT AND HEATER LOCATIONS
C
      READ(5,100) NHP,NSP
  100 FORMAT(2I5)
      READ(5,110) (IHP(I),I=1,NHP)
  110 FORMAT(16I5)
      READ(5,120) (ISP(I),I=1,NSP)
  120 FORMAT(16I5)
      WRITE(6,273)
  273 FORMAT('1')
      WRITE(6,259)
  259 FORMAT(29H INDICES OF HEAT PATCH POINTS//4X,1HI,2X,3HIHP)
      DO 260 L1=1,NHP
  260 WRITE(6,261) L1,IHP(L1)
  261 FORMAT(2I5)
      WRITE(6,262)
  262 FORMAT(25H INDICES OF SAMPLE POINTS//4X,1HI,2X,3HISP)
      DO 263 L1=1,NSP
  263 WRITE(6,261) L1,ISP(L1)
C
C     INPUT MATRIX A AND COMPUTE REDUCED MATRIX
C
      READ(4,130)((A(I,J),I=1,180),J=1,168)
  130 FORMAT(6E13.8)
C     SUBTRACT OUT SPHERICAL PART OF A(I,J)
C
      DIMENSION THET(15),BE(15),Y(15)
      FIM1=IM-1
      FKM=KM
      RF=2.0*DO*FNO
      SI=.5*DI/RF
```

```
      CI=SQRT(1.0-SI*SI)
      SO=.5*DO/RF
      CO=SQRT(1.0-SO*SO)
      THETI=ATAN(SI/CI)
      THETO=ATAN(SO/CO)
      DTHET=(THETO-THETI)/FIM1
      THETO=THETO-DTHET
      B2=0.
      DO 39 I=1,IM
      THET(I)=THETI+(I-1)*DTHET
      BE(I)=COS(THET(I))/COS(THETO)-1.
   39 B2=B2+BE(I)**2
      B2=B2*FKM
      DO 61 J=1,168
      WB=0.
      DO 62 K=1,KM
      DO 62 I=1,IM
      I1=(K-1)*IM+I
   62 WB=WB+A(I1,J)*BE(I)/B2
      DO 63 K=1,KM
      DO 63 I=1,IM
      I1=(K-1)*IM+I
   63 A(I1,J)=A(I1,J)-WB*BE(I)
   61 CONTINUE
   64 DO 150 I=1,NSP
      DO 150 J=1,NHP
      IS=ISP(I)
      JH=IHP(J)
  150 A(I,J)=A(IS,JH)
  152 FORMAT(10E13.8)
C
C     TO COMPUTE   I-A(1/ATA)AT
C
      DO 200 I=1,NHP
      DO 200 J=1,NHP
      DO 200 K=1,NSP
  200 B(I,J)=B(I,J)+A(K,I)*A(K,J)*BN
      DO 198 L1=1,NHP
      DO 198 L2=1,NHP
  198 D(L1,L2)=B(L1,L2)/BN
      CALL INVERT (B,NHP,50)
      DO 199 L1=1,NHP
      DO 199 L2=1,NHP
  199 B(L1,L2)=  BN *B(L1,L2)
      DO 197 I=1,NHP
      DO 197 J=1,NHP
      DO 197 K=1,NHP
  197 C(I,J)=C(I,J)+B(I,K)*D(K,J)
      WRITE(6,297)
  297 FORMAT('1')
      WRITE(6,152) ((C(I,J),I=1,10),J=1,10)
      DO 195 I=1,NHP
      DO 195 J=1,NHP
  195 C(I,J)=0.0
      DO 250 I=1,NHP
      DO 250 J=1,NSP
      DO 250 K=1,NHP
```

```
  250 C(I,J)=C(I,J)+B(I,K)*A(J,K)
C
C     COMPUTE WD
C
      DO 999 LL=1,5
      RS=RI+(RO-RI)*(IM-2)/(IM-1)
      DO 500 I=1,IM
      DO 500 J=1,KM
      K=IM*(J-1)+I
      R=RI+(RO-RI)*(I-1)/(IM-1)
      TH=0.5235988*(J-1)
      ARG=3.1415927*R/RS
      PY=R*SIN(TH)
      X=R*COS(TH)
      IF(LL.EQ.1) WD(K)=COS(ARG/2.)
      IF(LL.EQ.2) WD(K)=((RS-X)*(X+RS/2))/RS**2
      IF(LL.EQ.3) WD(K)=SIN(ARG)**2
      IF(LL.LE.3) GO TO 500
      IF(LL.EQ.4) XO=RS/2.
      IF(LL.EQ.5) XO=RS/(-2.)
      R1=SQRT((X-XO)**2+PY**2)
      WD(K)=1+COS(3.1415927*R1*4./RS)
      IF(R1.GE.RS/4.) WD(K)=0.
  500 CONTINUE
C
C     SUBTRACT OUT SPHERICAL PART FROM WD
C
      WB=0.0
      DO 300 IS=1,NSP
      JS=ISP(IS)
      I=ISP(IS)/IM
      I=ISP(IS)-I*IM
  300 WB=WB+WD(JS)*BE(I)
      DO 301 I=1,IM
  301 Y(I)=BE(I)*WB/B2
      DO 302 IS=1,NSP
      JS=ISP(IS)
      I=ISP(IS)/IM
      I=ISP(IS)-I*IM
  302 WD(JS)=WD(JS)-Y(I)
      IKM=IM*KM
C     WRITE(6,264)
C 264 FORMAT(12H DISTURBANCE)
C     WRITE(6,265)(WD(K),K=1,IKM)
C 265 FORMAT(15E8.3)
      DO 550 K=1,NSP
      IS=ISP(K)
  550 WD(K)=WD(IS)
C
C     COMPUTE PERFORMANCE INDEX
C
      AJ1=0.0
      DO 600 I=1,NSP
  600 AJ1=AJ1+WD(I)**2
      DO 620 I=1,NSP
  620 WW(I)=0.0
      DO 700 I=1,NSP
```

```
      DO 700 J=1,NSP
      DIJ=0.0
      IF(I.EQ.J) DIJ=1.0
      DO 701 K=1,NHP
  701 DIJ=DIJ-A(I,K)*C(K,J)
  700 WW(I)=WW(I)+DIJ*WD(J)
      AJ2=0.0
      DO 750 I=1,NSP
  750 AJ2=AJ2+WD(I)*WW(I)
      WRITE(6,1000) AJ1,AJ2
 1000 FORMAT(//46H THE PERFORMANCE INDEX BEFORE COMPENSATION IS ,E10.5,
     1//45H THE PERFORMANCE INDEX AFTER COMPENSATION IS ,E10.5)
      BJ1=SQRT(ABS(AJ1/NSP))
      BJ2=SQRT(ABS(AJ2/NSP))
      WRITE(6,1050) BJ1,BJ2
  999 CONTINUE
 1050 FORMAT(20H RMS ERROR BEFORE = ,E20.8/
     A       20H RMS ERROR AFTER  = ,E20.8)
      STOP
      END
```